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A New Uncertainty Measure and Application to Image Processing

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Abstract— Uncertainty measures form essential constituents of information theory as they provide a sufficient mechanism for determining the quantity of useful information contained in a system. In the present work, the concept of divergence between fuzzy sets are made use of in defining new measures of uncertainty in the framework of fuzzy rough sets. Further, these measures are utilized in developing an algorithm for binary image segmentation of a grey level image. Moreover, the proposed algorithm is implemented using different test images with the help of an OCTAVE program.

Keywords**—**Divergence, Image segmentation, Fuzzy Set, Rough set, Uncertainty measure.

I. INTRODUCTION

Claude E. Shannon put forward the theory of information in 1904 in order to lay the mathematical foundations for the systems that process and communicate information [1]. It has been a challenging task to tackle the difficulties arising out of the presence of noise, ambiguity and uncertainty in any information processing system. Different measures of uncertainty are proven to be very effective tools in handling this aspect.

Zdzislaw Pawlak's rough set theory and Lofti A. Zadeh's fuzzy set theory are two different approaches to address the problem of uncertainty and incompleteness in data at hand [2],[3]. The complementary nature of the two theories prompted the introduction of the hybrid theory called fuzzy rough set theory [4],[5]. Different measures of uncertainty are defined using the concepts in fuzzy set theory, rough set theory and also in fuzzy rough set theory [6],[7], [8],[9]. Segmentation of an image refers to the process of fragmenting the input image into a number of segments [10]. It helps in analyzing and understanding the image under consideration in a better way and extracting useful results out of it. Turning a grey level image to a binary image is termed as binary image segmentation. There are several existing image segmentation techniques which make use of the gray level histogram, spatial details, mathematical morphology, fuzzy set theoretic approaches, neural networks etc [11],[12],[13]. Thresholding techniques are very efficient methods which are widely used for binary image segmentation [14]. Of them, OTSU and FCM methods are the two very popular and classic techniques used for object background segmentation [15],[16]. The OTSU method determines the threshold by the maximization of the intensity variance between different classes, while the FCM method decides the threshold by the optimization of an objective function based on a measure of similarity between the pixels and the cluster centres. In the present paper, the divergence measure which quantifies the magnitude of dissimilarity between two given fuzzy sets is utilized to propose a new measure of uncertainty in the context of fuzzy approximation spaces. Based on the proposed measure, an algorithm for converting a grey level image to a binary image is also presented. The success of the algorithm is examined using many test images applying an OCTAVE program.

The structure of the paper is as follows: section II recalls some of the basic results and definitions, section III describes the new measure of uncertainty and investigate its properties, section IV presents the image segmentation algorithm and the experimental results and section V provides the conclusion.

II. RELATED WORK

This section recalls some of the basic definitions and concepts in connection with the theory of fuzzy rough sets. Throughout this paper, X represents a nonempty finite set of objects and $\mathcal{R}$ denotes a fuzzy equivalence relation on X.

A. Fuzzy Set Theory

The *fuzzy equivalence classes* of $\mathcal{R}$ are defined $\forall\, x \in X$ as, $\mu_{[x]_{\mathcal{R}}}(y) = \mathcal{R}(x, y),\ \forall\, y \in X$ *[17].*
The *fuzzy cardinality* of a fuzzy subset A of X is given by, $|A| = \sum_{x\in X} A(x)$ *[17].*

Let $\mathcal{F}(X)$ represents the collection of all the fuzzy subsets of X. A function $\delta: \mathcal{F}(X) \times \mathcal{F}(X) \longrightarrow [0,1]$ is termed as a *divergence measure* on $\mathcal{F}(X)$ if and only if for all fuzzy sets $A, B, C \in \mathcal{F}(X)$, $\delta(A, B) = \delta(B, A)$, $\delta(A, A) = 0$ and $\max\{\delta(A \cup C, B \cup C), \delta(A \cap C, B \cap C)\} \leq \delta(A, B)$ *[18].*
For the fuzzy subsets A, B, C, F of X,

$$A \subseteq B \subseteq C \subseteq F \Longrightarrow D(B,C) \leq D(A,F) \quad (1)$$

B. Fuzzy rough sets

Let R be a crisp equivalence relation on X. Then, *the lower* and *the upper approximations* of a crisp set $A \subseteq X$ are defined as

$$\underline{R}(A) = \{x \in X : [x]_R \subseteq A\} \quad (2)$$

$$\overline{R}(A) = \{x \in X : [x[_R \cap A \neq \varphi\} \quad (3)$$

respectively [19].

A *weak fuzzy partition* on X is the family of fuzzy sets $\{A_1, A_2, \ldots, A_k\}$ on X such that $\inf_x (\max_i A_i(x) > 0$ and $\sup_x (min\left(A_i(x), A_j(x)\right) < 1, \forall\, i,j$ [20].

Consider a nonempty set X of objects and a fuzzy equivalence relation $\mathcal{R}$ defined on X. The pair $(X,\mathcal{R})$ is said to be a *fuzzy approximation space*. The *fuzzy rough lower and upper approximations* of a fuzzy set A on X are defined by Dubois and Prade as

$$\mu_{\underline{\mathcal{R}}(A)}(x) = inf_{y \in X}\{\max[1-\mathcal{R}(x,y), \mu_A(y)]\} \quad (4)$$

$$\mu_{\overline{\mathcal{R}}(A)}(x) = sup_{y \in X}\{\min[\mathcal{R}(x,y), \mu_A(y)]\} \quad (5)$$

respectively [20].

Further generalizations of fuzzy rough approximations have been proposed by many authors *[21],[22],[23],[24]*.

C. Uncertainty Measures

In a crisp approximation space (X,R), the R - *roughness* of a set $A \subseteq X$ is given by $\rho_R(A) = 1 - \frac{|\underline{R}(A)|}{|\overline{R}(A)|}$ [19].

Consider $X/R = \{R_1, R_2, \ldots, R_k\}$. Then,

$$E(R) = \sum_{i=1}^{k} \frac{|R_i|}{|X|}\left(1 - \frac{|R_i|}{|X|}\right) \quad (5)$$

is called the *information entropy* of the approximation space (U,R) [25].

Let $X = \{x_1, x_2, \ldots, x_n\}$. Then,

$$H(\mathcal{R}) = \sum_{i=1}^{n} \frac{1}{|X|}\left(1 - \frac{|[x_i]_{\mathcal{R}}|}{|X|}\right) \quad (6)$$

is called the *complement information entropy* of the fuzzy approximation space $(X,\mathcal{R})$ [26].

III. FUZZY ROUGH UNCERTAINTY MEASURE

Take $X = \{x_1, x_2, \ldots, x_n\}$ and $\mathcal{R}$ be a fuzzy equivalence relation on X. Let $D(A,B)$ be a fuzzy divergence measure defined on $\mathcal{F}(X) \times \mathcal{F}(X)$ satisfying $D(A,B) = D(B,A)$, for all $A, B \in \mathcal{F}(X)$.

Definition 3.1: The *divergence-based information entropy* of the fuzzy approximation space $(X,\mathcal{R})$ is given to be

$$E_D(\mathcal{R}) = \frac{1}{|X|}\sum_{i=1}^{n} \frac{D(\hat{X},[x_i]_{\mathcal{R}})}{D(\hat{X},\hat{\varphi})} \quad (7)$$

where $\hat{\varphi}$ and $\hat{X}$ are the fuzzy sets corresponding to the empty set and the universal set respectively. Also, $[x_i]_{\mathrm{R}}$ is the fuzzy set on X given by $\mu_{[x_i]_{\mathrm{R}}}(x_j) = \mathcal{R}(x_i,x_j)$ for all $x_j \in X$.

Theorem 3.2: If $D(A,B) = \sum_{j=1}^{n}\left|A(x_j) - B(x_j)\right|$ and (X,R) is a crisp approximation space, then $E_D(R) = E(R)$.

Proof: Let $X/R = \{R_1, R_2, \ldots, R_k\}$. Then, every element x of X belongs to only one among the equivalence classes.

It follows that $R\,(x_i,x_j) = \begin{cases} 1, & if\ x_j \in [x_i]_R \\ 0, & otherwise \end{cases}$.

So, $D(\hat{X},[x_i]_R) = \sum_{j=1}^{n}\left|\hat{X}(x_j) - [x_i]_R(x_j)\right|$
$= \sum_{j=1}^{n}\left|1 - R\,(x_i,x_j)\right|$
$= \sum_{x_j \notin [x_i]_R} 1$
$= |([x_i]_R)^C|$
$= |X| - |[x_i]_R|$

Again, $D(\hat{X},\hat{\varphi}) = \sum_{j=1}^{n}\left|\hat{X}(x_j) - \hat{\varphi}(x_j)\right|$
$= \sum_{j=1}^{n}|1-0| = |X|$

Thus, $E_D(R) = \frac{1}{|X|}\sum_{i=1}^{n}\frac{D(\hat{X},[x_i]_{\mathrm{R}})}{D(\hat{X},\hat{\varphi})}$
$= \frac{1}{|X|}\sum_{i=1}^{n}\frac{|X| - |[x_i]_R|}{|X|}$
$= \sum_{i=1}^{n}\frac{1}{|X|}\left(1 - \frac{|[x_i]_R|}{|X|}\right)$

Denote $[x_i]_{\mathrm{R}}$ by R_j. Then, each value $\frac{1}{|X|}\left(1 - \frac{|[x_i]_R|}{|X|}\right)$ will be repeated exactly R_j times in the above summation.

Therefore, $E_D(R) = \sum_{j=1}^{k}\frac{|R_j|}{|X|}\left(1 - \frac{|R_j|}{|X|}\right)$
$= E(R)$

Theorem 3.3: If $D(A,B) = \sum_{j=1}^{n}\left|A(x_j) - B(x_j)\right|$ and $(X,\mathcal{R})$ is a fuzzy approximation space, then $E_D(\mathcal{R}) = H(\mathcal{R})$.

Proof: We have, $E_D(\mathcal{R}) = \frac{1}{|X|}\sum_{i=1}^{n}\frac{D(\hat{X},[x_i]_{\mathcal{R}})}{D(\hat{X},\hat{\varphi})}$
$= \frac{1}{|X|}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}|\hat{X}(x_j) - [x_i]_{\mathcal{R}}(x_j)|}{\sum_{j=1}^{n}|\hat{X}(x_j) - \hat{\varphi}(x_j)|}$
$= \frac{1}{|X|}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}|1-\mathcal{R}(x_i,x_j)|}{\sum_{j=1}^{n}|1-0|}$
$= \frac{1}{|X|}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}|1| - \sum_{j=1}^{n}|\mathcal{R}(x_i,x_j)|}{|X|}$, as $0 \leq \mathcal{R}(x_i,x_j) \leq 1, \forall\, i,j$
$= \frac{1}{|X|}\sum_{i=1}^{n}\frac{|X| - [x_i]_{\mathcal{R}}}{|X|}$. as $[x_i]_{\mathcal{R}} = \sum_{j=1}^{n}\left|\mathcal{R}(x_i,x_j)\right|$
$= \sum_{i=1}^{n}\frac{1}{|X|}\left(1 - \frac{[x_i]_{\mathcal{R}}}{|X|}\right)$
$= H(\mathcal{R})$

Theorem 3.4: If $\mathcal{R}_1$ and $\mathcal{R}_2$ are two fuzzy equivalence relations on X, then $\mathcal{R}_1 \subseteq \mathcal{R}_2 \Longrightarrow E_D(\mathcal{R}_1) \geq E_D(\mathcal{R}_2)$.

Proof: We have,

$\mathcal{R}_1 \subseteq \mathcal{R}_2 \Longrightarrow \mathcal{R}_1(x_i,x_j) \leq \mathcal{R}_2(x_i,x_j), \forall x_i, x_j \in X$

$\Rightarrow [x_i]_{\mathcal{R}_1}(x_j) \leq [x_i]_{\mathcal{R}_2}(x_j) \leq 1, \forall x_i, x_j \in X$
$\Rightarrow [x_i]_{\mathcal{R}_1} \subseteq [x_i]_{\mathcal{R}_2} \subseteq \hat{X}, \forall x_i \in X$
$\Rightarrow D(\hat{X}, [x_i]_{\mathcal{R}_1}) \geq D(\hat{X}, [x_i]_{\mathcal{R}_2})$
$\Rightarrow \frac{D(\hat{X}, [x_i]_{\mathcal{R}_1})}{D(\hat{X}, \hat{\varphi})} \geq \frac{D(\hat{X}, [x_i]_{\mathcal{R}_2})}{D(\hat{X}, \hat{\varphi})}$
$\Rightarrow \sum_{i=1}^{n} \frac{D(\hat{X}, [x_i]_{\mathcal{R}_1})}{D(\hat{X}, \hat{\varphi})} \geq \sum_{i=1}^{n} \frac{D(\hat{X}, [x_i]_{\mathcal{R}_2})}{D(\hat{X}, \hat{\varphi})}$
$\Rightarrow E_D(\mathcal{R}_1) \geq E_D(\mathcal{R}_2)$

Definition 3.5: The divergence based fuzzy rough uncertainty of a fuzzy set $A \in \mathcal{F}(X)$ with respect to $\mathcal{R}$ is defined as

$$E_D^{\mathcal{R}}(A) = \frac{D(\overline{\mathcal{R}}(A), \underline{\mathcal{R}}(A))}{D(\hat{X}, \hat{\varphi})} \tag{8}$$

Proposition 3.6: For all fuzzy sets $A \in \mathcal{F}(X)$ and for all divergence measures D, $0 \leq E_D^{\mathcal{R}}(A) \leq 1$.

Proof: Consider the fuzzy sets $\hat{\varphi}$, $\hat{X}$, $\underline{\mathcal{R}}(A)$, $\overline{\mathcal{R}}(A)$ on X. Clearly, $\hat{\varphi} \subseteq \underline{\mathcal{R}}(A) \subseteq \overline{\mathcal{R}}(A) \subseteq \hat{X}$. Therefore, by the property of divergence measure, $0 \leq D\left(\overline{\mathcal{R}}(A), \underline{\mathcal{R}}(A)\right) \leq D(\hat{X}, \hat{\varphi})$. Hence, $0 \leq \frac{D(\overline{\mathcal{R}}(A), \underline{\mathcal{R}}(A))}{D(\hat{X}, \hat{\varphi})} \leq 1$. Thus, $0 \leq E_D^{\mathcal{R}}(A) \leq 1$.

Theorem 3.7: If $\mathcal{R}_1$ and $\mathcal{R}_2$ are two fuzzy equivalence relations on X, then $\mathcal{R}_1 \subseteq \mathcal{R}_2 \Rightarrow E_D^{\mathcal{R}_1}(A) \geq E_D^{\mathcal{R}_2}(A)$.
Proof: $\mathcal{R}_1 \subseteq \mathcal{R}_2 \Rightarrow \mathcal{R}_1(x_i, x_j) \leq \mathcal{R}_2(x_i, x_j), \forall x_i, x_j \in X$
$\Rightarrow \mu_{\underline{\mathcal{R}_1}(A)}(x) \geq \mu_{\underline{\mathcal{R}_2}(A)}(x)$ and $\mu_{\overline{\mathcal{R}_1}(A)}(x) \leq \mu_{\overline{\mathcal{R}_2}(A)}(x)$
$\Rightarrow \underline{\mathcal{R}_1}(A) \supseteq \underline{\mathcal{R}_2}(A)$ and $\overline{\mathcal{R}_1}(A) \subseteq \overline{\mathcal{R}_2}(A)$
$\Rightarrow \underline{\mathcal{R}_2}(A) \subseteq \underline{\mathcal{R}_1}(A) \subseteq \overline{\mathcal{R}_1}(A) \subseteq \overline{\mathcal{R}_2}(A)$
$\Rightarrow D(\overline{\mathcal{R}_2}(A), \underline{\mathcal{R}_2}(A)) \geq D(\overline{\mathcal{R}_1}(A), \underline{\mathcal{R}_1}(A))$
$\Rightarrow \frac{D(\overline{\mathcal{R}_2}(A), \underline{\mathcal{R}_2}(A))}{D(\hat{X}, \hat{\varphi})} \geq \frac{D(\overline{\mathcal{R}_1}(A), \underline{\mathcal{R}_1}(A))}{D(\hat{X}, \hat{\varphi})}$
$\Rightarrow \sum_{i=1}^{n} \frac{(\overline{\mathcal{R}_2}(A), \underline{\mathcal{R}_2}(A))}{D(\hat{X}, \hat{\varphi})} \geq \sum_{i=1}^{n} \frac{D(\overline{\mathcal{R}_1}(A), \underline{\mathcal{R}_1}(A))}{D(\hat{X}, \hat{\varphi})}$
$\Rightarrow E_D^{\mathcal{R}_1}(A) \geq E_D^{\mathcal{R}_2}(A)$

IV. APPLICATION TO IMAGE PROCESSING

Let X denote the set of all pixel values in a grey level image of order $m \times n$ and $t(i,j)$ be the grey level value of pixel (i,j). Then, $X = \{0,1,\ldots,255\}$. As the first step, we divide the entire image into m parts and obtain the most frequent grey level value (say v_r) in each part. Corresponding to each v_r, for $r = 1,2,\ldots,N$, we define a fuzzy set A_r on X, as

$$A_r(i) = \frac{|t(i,j) - v_r|}{255}. \tag{9}$$

Then, $P = \{A_r : r = 1,2,\ldots,m\}$ will constitute a weak fuzzy partition on X. For each grey level value s, we define a fuzzy set corresponding to the object in the image as

$$F_s(i,j) = \begin{cases} \frac{t(i,j)}{255}, & if \quad t(i,j) > s \\ 0, & otherwise \end{cases} \tag{10}$$

Let $\underline{P}(F_s)$ and $\overline{P}(F_s)$ denote the fuzzy rough lower and upper approximations of F_s with respect to the weak fuzzy partition P. That is;

$$\underline{P}(F_s) = \sup_r \left\{ \min_{i,j} \left[A_r(i,j), \inf_{i,j}\left(max(1 - A_r(i,j), F_s(i,j)) \right) \right] \right\} \tag{11}$$

$$\overline{P}(F_s) = \sup_r \left\{ min_{i,j} \left[A_r(i,j), \sup_{i,j}\left(min(A_r(i,j), F_s(i,j)) \right) \right] \right\} \tag{12}$$

The fuzzy rough uncertainty measure corresponding to s is given by

$$E_D^{\mathcal{R}}(F_s) = \frac{D(\overline{\mathcal{R}}(F_s), \underline{\mathcal{R}}(F_s))}{D(\hat{X}, \hat{\varphi})} \tag{13}$$

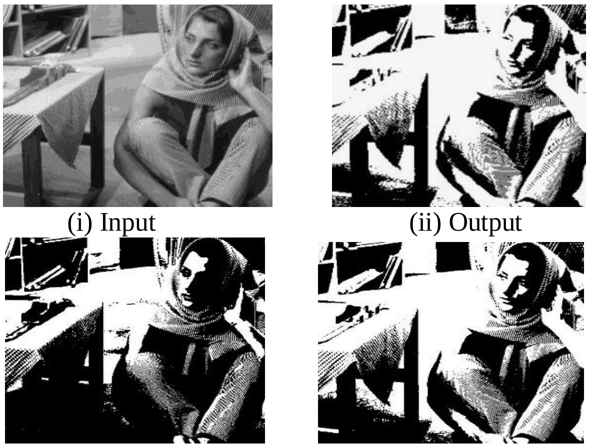
Here, we take, $D(A,B) = \sum_{j=1}^{n} |A(x_j) - B(x_j)|$.
The value of s at which the uncertainty measure becomes the minimum is chosen as the threshold c. In case the minimum value occurs at more than one value of s, the highest value among them is determined as c. Then the image is converted into the corresponding binary image.

Algorithm4.4: The algorithm for the proposed binary image segmentation process is

1. *Input the grey level image X*
2. *Obtain the corresponding weak fuzzy partition P*
3. *Determine the fuzzy set F_s for s=1:256 using equation (10)*
4. *Compute the fuzzy rough approximations of F_s using equations (11) and (12)*
5. *Calculate the fuzzy roughness measure $E_D^{\mathcal{R}}(F_s)$ using equation (13)*
6. *Obtain c, which is the value of s for which $E_D^{\mathcal{R}}(F_s)$ is the minimum*
7. *Convert the image into a binary image using c*

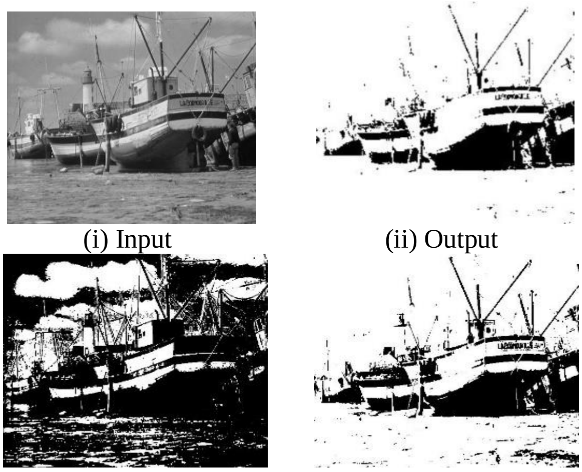
IV. EXPERIMENTAL RESULTS

The algorithm for object background segmentation is experimented with four different test images which are barbara, boat, bridge, lena and cameraman. The input images and the output images obtained using the proposed method, OTSU and FCM methods are given as figures 1to 5. One can easily observe that fine binary images are obtained by the use of the proposed method. The figures clearly convey that the overlapping of the foreground background pixels in the images segmented using the proposed method is lesser than those of the other two methods. However, in the case of barbara image, dominance of white pixels occurred on some portions. Yet, the image of the lady and the table are clearly separated from the other parts. The output images are compared using their root mean square error values (table 1) and it is found that the proposed method produces output images with lesser error in four out of five test images.



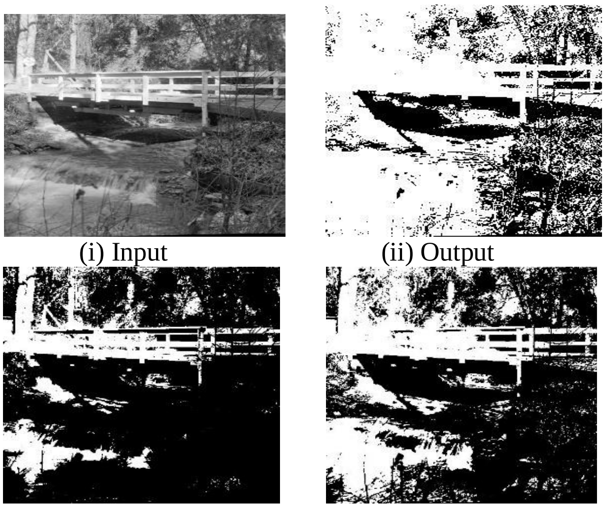
(i) Input (ii) Output (iii) FCM (iv) OTSU

Figure 1: Barbara



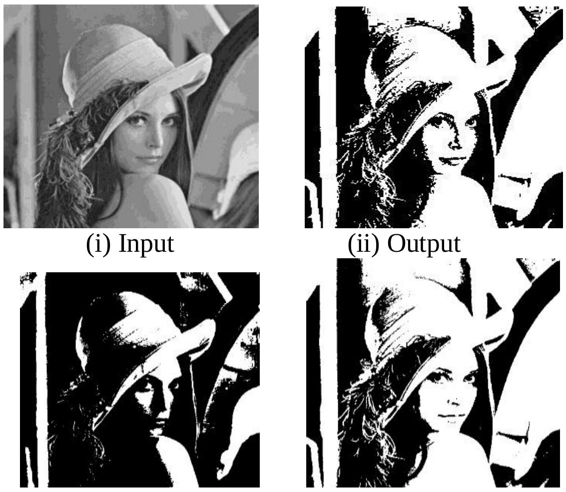
(i) Input (ii) Output (iii) FCM (iv) OTSU

Figure 2: Boat



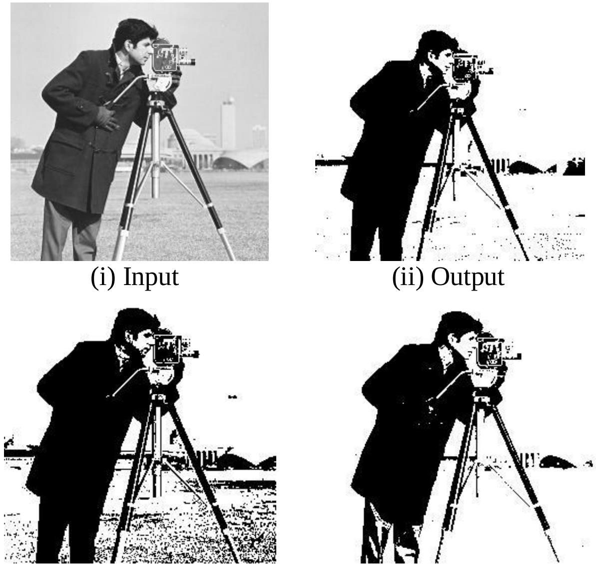
(i) Input (ii) Output (iii) FCM (iv) OTSU

Figure 3: Bridge



(i) Input (ii) Output (iii) FCM (iv) OTSU

Figure 4: Lena



(i) Input (ii) Output (iii) FCM (iv) OTSU

Figure 5: Cameraman

Table 1: Root Mean Square Error Values

Sl.No.	Image	Proposed Method	FCM Method	OTSU Method
1	Barbara	**10.190**	13.093	11.243
2	Boat	**5.776**	13.236	7.3552
3	Bridge	**8.0197**	14.045	12.535
4	Lena	10.505	13.768	**10.313**
5	Cameraman	**8.2662**	10.343	8.6094

V. CONCLUSION

Uncertainty measures are used widely in the process of information acquisition. They have been successfully applied in many image processing techniques. In this paper, new measures of uncertainty have been defined using the divergence measures of fuzzy sets in the context of fuzzy approximation spaces. Also, divergence based fuzzy rough uncertainty of a fuzzy set has been introduced. An object background segmentation technique using the proposed measure of uncertainty has been presented and experimented with different test images. The segmented images of five common test images have been compared with those of OTSU method and FCM method. It is observed that the overlapping of the foreground background pixels in the images segmented using the proposed method is lesser than those of the other two methods. Also, the root mean square error values of the output binary images obtained using the proposed method are found to be lesser than those of the other two methods in four out of five test images.

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