

A Study on γ - Optimization Parameter and Phase Margin of Fourth Order Phase-Locked Loop

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Abstract— A 4th order phase-locked loop model by considering two different filter configurations in the loop have been developed and behavioral simulation has been performed on MATLAB platform to study the impact of gamma-optimization parameter on it and also to study the stability of the system in terms of phase margin. We have also investigated the bandwidth and lock time of the system.

Keywords— Bandwidth, gamma optimization parameter, lock time, phase margin, stability.

I. INTRODUCTION

The concept of phase-locked loop (PLL) was first emerged in the early 1930's and nowadays PLL is one of the main building blocks in electronics technology due to the development of integrated circuits (ICs). A basic form of a PLL consists of four fundamental blocks. The phase detector (PD), loop filter (LF) and voltage controlled oscillator (VCO) are in forward path and frequency divider (FD) is in the feedback path as shown in **Figure 1**. In the synchronized state, the phase difference between the VCO output signal and the input reference signal is zero or a very small value and no frequency offset between the input and output signal exist. If there is a phase difference, the control signal of VCO in the PLL try to removes the frequency offset and minimizes the phase difference to a minimum [1, 2, 3].

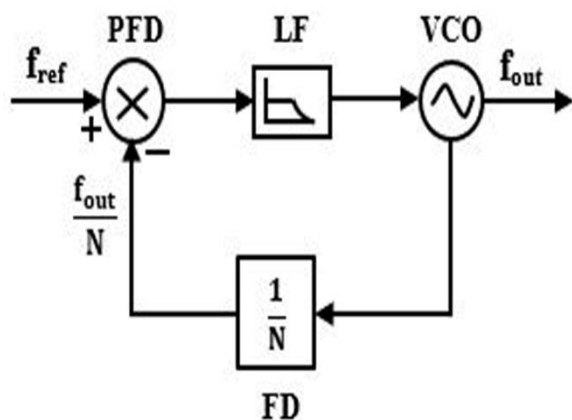


Figure 1. General block diagram of PLL

The PD produces a voltage which is proportional to phase difference between the reference and VCO output signal. The main function of the loop filter (LF) is to determine the loop dynamics of the system. The output of the LF is fed to the VCO as its tuning voltage. The frequency divider (FD) is used in the feedback loop to generate a frequency that is multiple of a reference frequency.

The gamma (γ)-optimization parameter is regarded as one of the most important parameters to design the LF of PLL. It maximizes the phase margin (PM) value at the loop bandwidth (BW). The γ value one is a good criterion for minimizing the lock time (LT) of the PLL for a given PM and loop BW. If we allow the γ other than one, it can improve the LT up to 30% depending on the value of PM. If γ value is slightly higher than one, it represents optimum value of PM at a frequency lower than the loop BW. If γ is lower than one, it will represent the optimum PM with a frequency higher than the loop BW [4, 7].

The section I and II of the paper contain the introduction and related work respectively. Section III explain the objective and methodology of the work. The theoretical estimation part and simulations are given in section IV and V respectively. Section VI describes the result and discussion part. Section VII contain the conclusion of the proposed model.

II. RELATED WORK

There are very few works related to γ - optimization parameters of PLL in literature. In 2006, Dean Banerjee had defined γ to design LF of PLL and he also studied the relationship between γ and PM [5]. In the year 2007, Lin Jia,

Kiat Seng Yeo Jian Guo Ma, Manh Anh Do and Xiao Peng Yu analyzed the stability of PLL in terms of γ and PM and found that the stability limit for γ is different with different PM for the system [6]. In the year 2013, K. Kalita and T. Bezboruah reported a PLL model to study how γ and PM impact on PLL for designing of frequency synthesizer [7].

The optimal choice of γ depends on PM of the system and hence, it is necessary to rigorously study about the γ and PM together. In this paper, we will present a 4th order PLL model and study about the impact of γ and PM on the system.

III. OBJECTIVE AND METHODOLOGY

The main objective of the proposed work is to model and simulate a 4th order PLL model to analysed the impact of γ and PM on the system. We have considered two different 3rd order active filter configurations in developing the PLL model. One with active lag – lead filter (ALLF) and the other with standard feedback approach (SFA). The behavioural simulation is performed on MATLAB platform to study the impact of: (i) γ , (ii) PM, (iii) BW, and (iv) LT on the system.

The methodology towards implementation of the work involves:

- Derivation of system transfer function (TF) of the model in s-domain by considering both LF approach in the loop.
- Derivation of two mathematical expressions for γ of the system by considering both the LF approach in the loop.
- Simulation of the derived γ of the system to study the various aspects of the system.

IV. THEORETICAL ESTIMATIONS

The functional block diagram of the proposed behavioural 4th order PLL with loop in ALLF and SFA are shown in **Figure 2** and **Figure 3** respectively.

A. Estimation of the TF of the system with loop in SFA

The system TF of the proposed system can be expressed as:

$$H(s) = \frac{\text{Forward Gain}}{1 + \text{Loop Gain}} \quad (1)$$

The forward gain and loop gain of the system can be expressed as:

$$\text{Forward Gain} = K_d F(s) A_d \frac{K_0}{s} \quad (2)$$

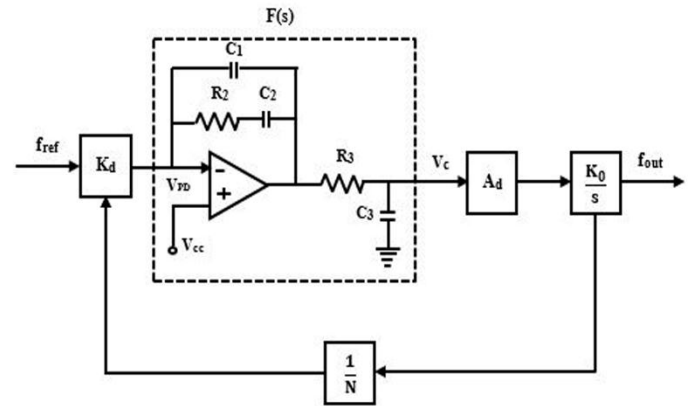


Figure 2. Functional block diagram of the system with SFA in the loop

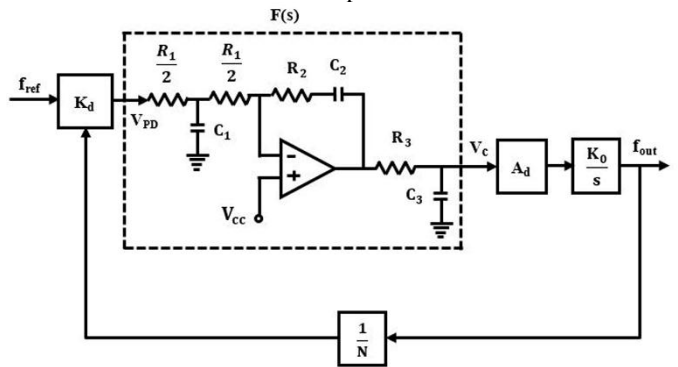


Figure 3. Functional block diagram of the system with ALLF in the loop

$$\text{Loop Gain} = \frac{K_d F(s) A_d K_0}{Ns} \quad (3)$$

Where, K_d = PFD gain in volt/radian, $F(s)$ = TF for LF, K_0/s = VCO gain in MHz/volt, A_d = amplifier gain, and N = division ratio.

The TF of the loop with SFA can be derived as:

$$F(s) = \frac{1 + sC_2R_2}{s^3 C_1 C_2 C_3 R_2 R_3 + s^2 \{C_1 C_2 R_2 + C_3 R_3 (C_1 + C_2)\} + s(C_1 + C_2)} \quad (4)$$

Now, by combining equations (1), (2), (3) and (4), the system TF of the system with loop in SFA can be simplified as:

$$H(s) = \frac{K_d K_0 A_d (1 + sC_2 R_2)}{s^4 C_1 C_2 C_3 R_2 R_3 + s^3 \{C_1 C_2 R_2 + C_3 R_3 (C_1 + C_2)\} + s^2 (C_1 + C_2) + (1 + sC_2 R_2) K A_d} \quad (5)$$

Where, loop gain constant $K = K_0 K_d / N$ [5].

B. Estimation of the TF of the system with ALLF in the loop

The TF of the ALLF filter section can be derived as:

$$F(s) = \frac{1 + sC_2R_2}{s^3 C_1C_2C_3R_1 \frac{R_1}{4} R_3 + s^2 (C_1C_2R_1 \frac{R_1}{4} + C_2C_3R_1R_3) + sC_2R_1} \quad (6)$$

By combining equations (1), (2), (3) and (6), the system TF of the PLL with ALLF in the loop can be simplified as:

$$H(s) = \frac{K_d K_0 A_d (1 + sC_2R_2)}{s^4 C_1C_2C_3R_1 \frac{R_1}{4} R_3 + s^3 (C_1C_2R_1 \frac{R_1}{4} + C_1C_2R_1R_3) + s^2 C_2R_1 + (1 + sC_2R_2)KA_d} \quad (7)$$

C. Estimation for γ

The γ - value can be determined in terms of system BW and time constants of the LF of the system [4, 5, 7]. The γ of the proposed system with loop in SFA can be derived as:

$$\gamma_{SFA} = (2 * \pi * BW_{SFA})^2 * T_2 * (T_1 + T_2) \quad (8)$$

Where, BW_{SFA} = Loop BW of the system with SFA approach; T_1, T_2, T_3 = Time constants of the SFA LF of the PLL model

and $T_1 = \frac{C_1C_2R_2}{C_1 + C_2}, T_2 = C_2R_2$ and $T_3 = C_3R_3$.

Similarly, for ALLF in the loop:

$$\gamma_{ALLF} = (2 * \pi * BW_{ALLF})^2 * T_2 * (T_1 + T_3 + T_4) \quad (9)$$

Where, BW_{ALLF} = Loop BW of the model with ALLF LF approach and T_1, T_2, T_3, T_4 = Time constants of the ALLF LF of the PLL and $T_1 = C_2R_1, T_2 = C_2R_2, T_3 = 0.25 * C_1R_1$ and $T_4 = C_3R_3$.

V. SIMULATIONS

We simulate the system TF and γ given in equation (5) and (8) for the system with SFA in the loop and equation (7) and (9) for the ALLF in the loop for the system to study the behaviour of the system and impact of γ and PM on it. The behavioural simulations have been performed by using MATLAB in s-domain. The bode plot analysis is used to simulate the closed loop TF of the system for analysis of γ , PM and loop BW. The γ is calculated from loop BW of the system and time constants of the LF used in the loop. The bode plot is an important tool which gives direct analysis of the loop BW and PM. **Figure 4(a)** and **Figure 4(b)** show the simulated responses for the γ of closed loop gain of the loop with SFA in the loop and ALLF in the loop respectively. The

x - axis represents the frequency in radian per second and y - axis represents the magnitude in dB. Figure 5(a) and Figure 5(b) show the PM analysis of the system with both the configuration.

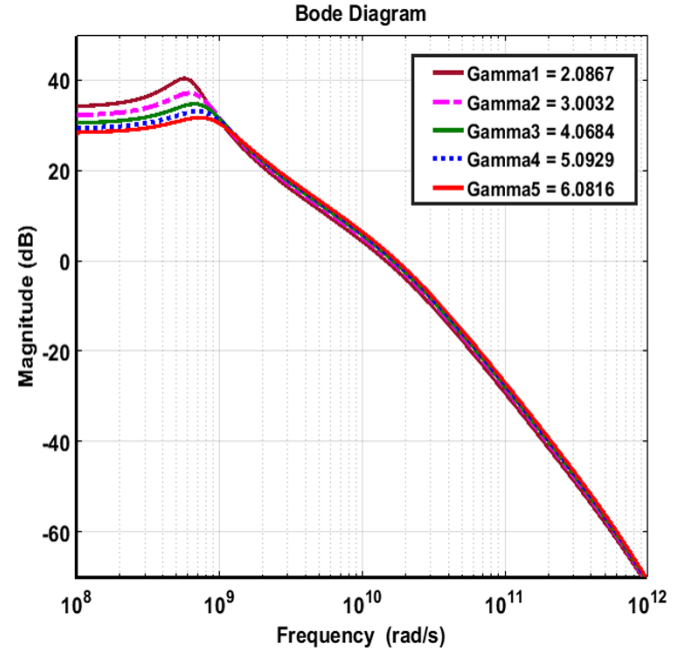


Figure 4(a). Impact of γ optimization parameter on closed loop gain of the model with SFA LF

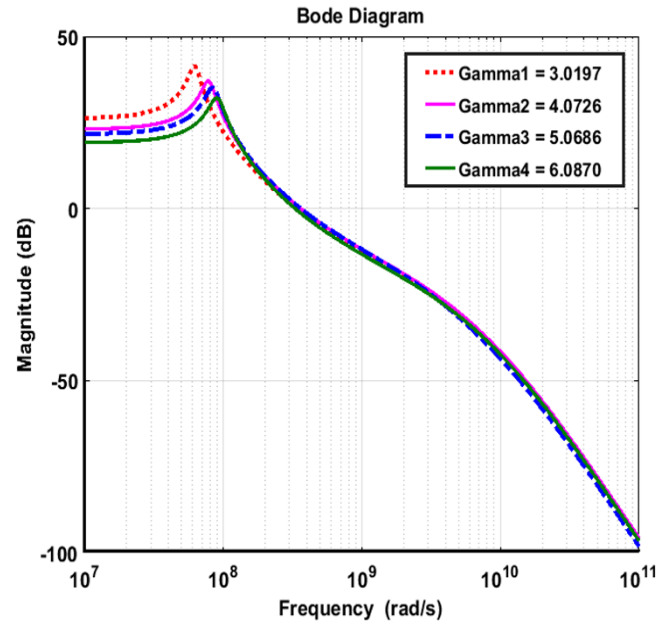


Figure 4(b). Impact of γ optimization parameter on closed loop gain of the model with ALLF LF

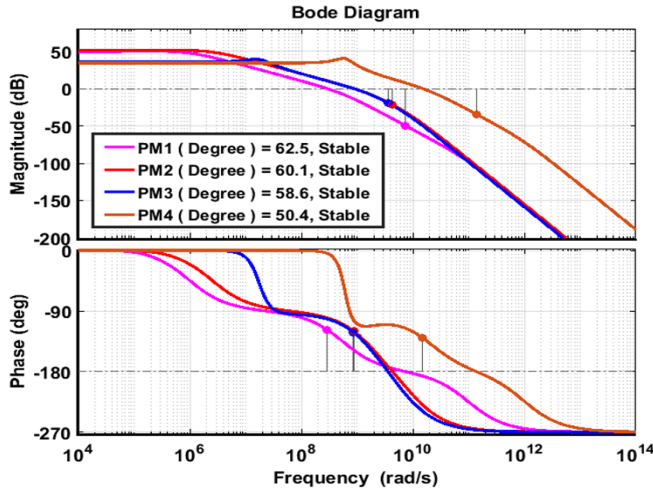


Figure 5(a). Simulated responses for PM of the model with SFA LF

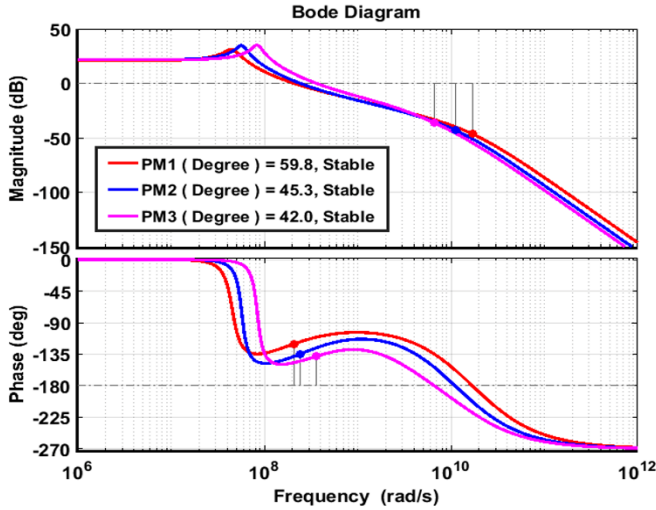


Figure 5(b). Simulated responses for PM of the model with ALLF LF

VI. RESULTS AND DISCUSSION

The simulation results obtained for γ , PM, LT and BW of the system for different test cases are given in Table 1 and Table 2. From Figure 4(a) and Figure 4(b), it is observed that the peak of the closed loop gains for both the LF sections of the system decreases as γ is increased. It is also observed that the system with SFA gives lower peak and more flatness as compared to loop with ALLF. It is also observed from the table that the results are better than the results presented by other research group [7]. The PM is another significant parameter of PLL which determines the stability of the system. To achieve more stability and to reduce the amount of peaking in the LF response and the RMS phase error a higher value of PM is required [5]. From Figure 5(a) and Figure 5(b) it is observed that if the PM is increasing the peak of the closed loop gain decreases and the system become more stable.

We have also plotted the relationship of γ between PM, LT and BW as shown in Figure 6, Figure 7 and Figure 8 for both the approaches of the system. From Figure 6 and Figure 7, it is observed that γ is inversely proportional to both PM and LT of the system. It is also observed from Figure 8 that BW of the system is linearly increases with γ .

Table 1. Simulated results for different parameters of the model with SFA

SL NO.	BW (MHz)	PM (Degree)	LT (us)	γ - value
1	0.1907	62.5	3.3378	1.0158
2	0.4706	60.1	1.3528	1.0553
3	0.9659	60.1	0.6590	1.0733
4	1.3783	60.0	0.4618	1.0944
5	1.6206	60.0	0.3928	1.1229
6	1.7939	59.2	0.3548	1.2267
7	4.9408	58.6	0.1288	1.3153
8	21.907	56.3	0.0290	1.5232
9	57.190	52.4	0.0111	1.7051
10	159.88	50.4	0.0039	1.9760
11	159.99	49.2	0.0039	2.0867
12	185.24	48.4	0.0034	3.0032
13	210.76	47.8	0.0030	4.0684
14	231.76	47.4	0.0027	5.0929
15	251.71	47.4	0.0025	6.0816

Table 2. Simulated results for different parameters of the model with SFA

SL NO.	BW (MHz)	PM (Degree)	LT (us)	γ -value
1	4.3458	65.8	0.1464	1.0152
2	11.453	59.8	0.0555	1.0598
3	12.334	52.1	0.0516	1.0775
4	12.524	50.1	0.0508	1.0968
5	12.628	49.9	0.0504	1.1066
6	13.015	48.4	0.0489	1.2085
7	13.410	47.0	0.0474	1.3023
8	14.350	45.3	0.0443	1.5206
9	15.278	43.1	0.0416	1.8069
10	15.310	42.9	0.0415	1.9068
11	15.314	42.8	0.0415	2.0918
12	15.470	42.4	0.0411	3.0197
13	19.607	42.2	0.0324	4.0726
14	21.135	42.0	0.0301	5.0686
15	22.701	41.3	0.0280	6.0870

Table 3. Comparison table of LT for different value of γ and PM

Parameter	Ref [5]	Ref [7]	This work	
			ALLF LF	SFA LF
LT (μ s)	232.4 – 246.4	0.091176 – 0.26264	0.0280 – 0.1464	0.0025 – 3.3378

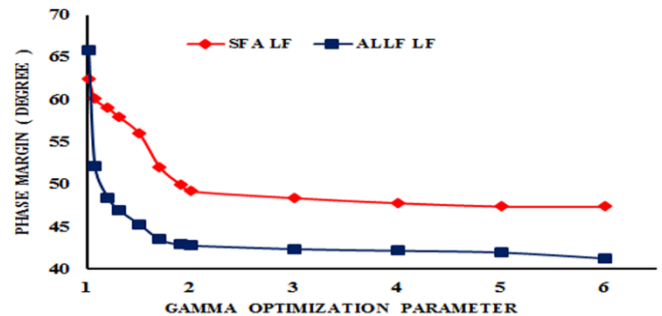


Figure 6. Responses for PM versus γ – optimization parameter

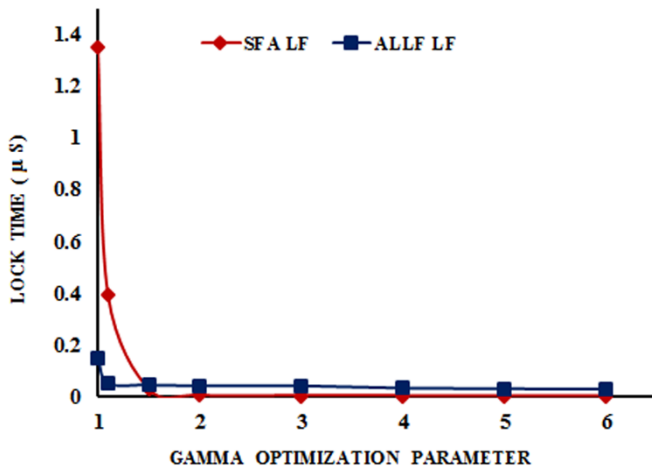


Figure 7. Responses for LT versus γ – optimization parameter

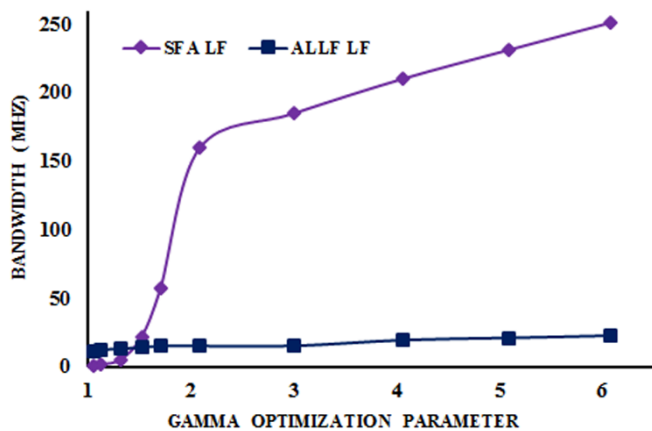


Figure 8. Response for BW versus γ – optimization parameter

VII. CONCLUSION

From the analysis of the proposed system, we can conclude that if we increase γ , the peak of the response of LF of the system reduces marginally. Though, we assume that the γ value is one for LF design, but from the present study, it is clear that there is further room for optimization. The fastest LT of the system is obtained to be $0.0025\mu\text{s}$ for $\gamma = 6.0816$ with $\text{PM} = 47.4^\circ$. The LT of the system depends on γ and PM, so we have to choose PM and γ value in such a way that the LT can be minimized. For a stable LF design, the PM must be higher than 20° and lower than 80° [5] which is fulfilled by the proposed system. It is also observed that the flatness of the closed loop gain is increased as PM increases and hence the stability of the system is also increased.

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Authors Profile

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