

Performance Analysis of Turbo Codes Using CRC and FC with OFDM

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Available online at: www.ijcseonline.org

Received: 23/Nov/2017, Revised: 30/Nov/2017, Accepted: 21/Dec/2017, Published: 31/Dec/2017

Abstract— Long Term Evolution (LTE) is an evolving wireless communication standard that requires high data rate, higher bandwidth efficiency and better coverage. So, spectrally efficient techniques like Orthogonal Frequency Division Multiplexing (OFDM), Multiple Input Multiple Output and Error Control Codes with high coding gain are constituents of LTE. OFDM partitions a channel with wide bandwidth into several channels with narrow bandwidth that are flat, in order to mitigate the effects of multipath fading and Inter Symbol Interference (ISI). Error control codes enhance reliable transmission over noisy fading channels. The objective of this work is to increase data rate and reduce bit error rate by combining OFDM and Turbo Codes. Bit error rate is further reduced by applying Cyclic Redundancy Check (CRC) and Flip and Check algorithm (FC) along with turbo codes. In this paper, MAP and MAX Log MAP algorithms have been applied in the turbo decoding process and their performance compared. The proposed algorithm has been implemented and simulated using MATLAB.

Keywords—Turbo Codes, MAP, MAX Log MAP, Cyclic Redundancy Check, Flip and Check, OFDM, LTE

I. INTRODUCTION

A communication system is used for transmission of information from one point to another through a channel. Digital communication has grown tremendously by leaps and bounds over several years. In digital communication, the information to be transmitted is represented in binary form. The binary bits are modulated and transmitted through a channel. The channel introduces noise and due to its interference, it will corrupt the transmitted data. This causes errors in the received signal. At the receiver side, the received signal is converted back to binary, and the bits are decoded to reconstruct the original message.

Communication can be through wired or wireless channels. Today, most of the activities involve wireless communication in one way or other. The wireless channels are not as reliable as wired channels due to additive noise and fading effects. The errors introduced into the transmitted bits are more in wireless communication than in wired communication. Hence, all wireless communications require advanced error detection and correction techniques in order to reduce bit error rate, enhance reliable transmission and improve Quality of Service (QoS). The number of errors in the transmitted bit stream depends on the noise and interference present in the channel. These errors can be detected and corrected using channel coding techniques.

Error Detection and Correction is collectively known as Error Control Coding. Error correction codes introduce coding gain

which is the difference in Signal-to-Noise Ratio (SNR) at which a particular Bit Error Rate (BER) can be achieved with and without error control coding. The different types of Error Control Codes are Linear Block Codes, Cyclic Codes, Convolution Codes, Reed Solomon Codes, Turbo Codes, Low Density Parity Check Codes, etc. Turbo codes are error correcting codes that achieve near Shannon limit. However, time latency and computational complexity are the major concerns from the implementation point of view. Researchers have explored the possibilities to reduce latency by applying proper termination for the turbo decoding algorithm based on its convergence properties [1].

In this paper, turbo codes have been incorporated with Cyclic Redundancy Check (CRC) and Flip and Check (FC) algorithms to improve the error performance of the system. This also decreases the time latency and computational complexity in the turbo encoding / decoding process. In Orthogonal Frequency Division Multiplexing (OFDM) several data channels can be simultaneously transmitted through band-limited channels at high data rates without interchannel and intersymbol interferences. A high data rate bit-stream is split into multiple low data rate bit-streams and each stream is modulated using different sub-carriers that are orthogonal to each other. The combination of turbo codes, CRC, FC and OFDM gives lower BER in wireless channels with additive noise and fading effects.

Rest of the paper is organized as follows. Section II contains the related work. Section III describes the turbo encoder,

decoder and interleaver used in the system. Section IV gives the CRC and Flip and Check algorithm. Section V describes the OFDM technique. The proposed communication system is explained in Section VI and the results are discussed in Section VII. The paper concludes with Section VIII.

II. RELATED WORK

Turbo codes are the most powerful error detecting and correcting codes, developed by S C. Berrou, A. Glavieux, P. Thitimajshima in 1993 [2, 3]. The performance of turbo codes over noisy channels is close to the Shannon limit. This increases the quality of service and transmission can take place at higher data rates. Many of the wireless standards based on turbo codes are rated based on the complexity, delay, memory requirements and signal to noise ratio. The turbo encoder consists of several Recursive Systematic Convolutional Encoders (RSC) connected in parallel, where input to the encoders may be from the data source or from interleaver [4]. If the interleaver length is increased the turbo code performance can be improved. Many algorithms have been proposed for turbo decoding but the time and computational complexity make it difficult for implementation. Interleaver design for serially concatenated convolutional codes with reduced complexity and improved performance has been proposed in [5]. In [6] optimum interleaver design for mobile communication applications have been proposed to reduce the time latency. An interleaver design has been proposed to optimize the convergence time of the turbo decoder in [7]. Conventional Maximum Likelihood (ML) decoding algorithm is compared with list decoding algorithm in [8]. List decoding of turbo codes [9] based on List Output Viterbi Algorithm (LOVA) has been proposed and its performance has been compared with the ML list decoder. In [10], Cyclic Redundancy Check and Flip and Check Algorithm have been incorporated in the turbo encoding / decoding process to decrease the time latency and computational complexity.

Orthogonal Frequency Division Multiplexing achieves high data rate transmission over multipath fading channels [11]. The performance of different turbo decoding algorithms with OFDM over wireless channels is analyzed in [12]. If it has multiple input and multiple output antennas at both transmitting and receiving side, it will improve the channel capacity, coverage and reliability of transmission. OFDM combined with recursive turbo decoding, achieves performance close to Shannon limit in the turbo error floor region [13, 14]. 3GPP LTE [15] employs OFDM for uplink and OFDMA for downlink with modulation techniques such as QPSK and QAM. It also applies Multiple Input Multiple Output (MIMO) to achieve robustness and high data rates [16].

III. TURBO CODING

A. Turbo Encoder

The turbo encoder used in this work is specified in the Long Term Evolution (LTE) standard, shown in Figure 1 [17]. It

consists of two Recursive Systematic Convolutional (RSC) encoders, with one input applied as it is and the other one applied through an interleaver. The code rate is 1/3. Here, the choice of the interleaver is pseudo-random which reorders the input data bits in a random manner. The constraint length of the LTE turbo encoder is $K=4$, each encoder being identical. The block length is varied from 40 to 6144 bits. The first turbo encoder receives the information bit c_k , the encoded output of the first RSC encoder is z_k . The input information becomes scrambled by the interleaver, the interleaved data sequence c'_k is given to the input of second RSC encoder. The second encoder output becomes z'_k . The encoded sequence consisting of u_k, z_k, z'_k are transmitted over the channel to the receiver.

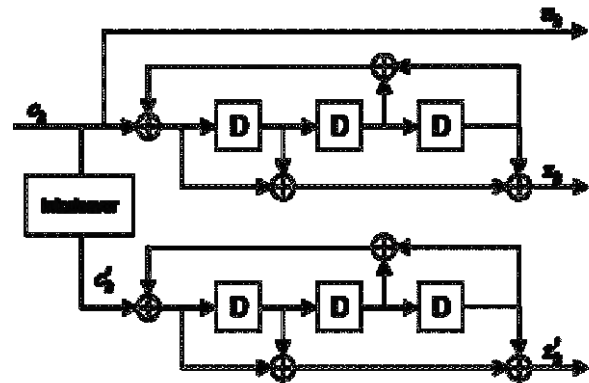


Figure 1. LTE Turbo Encoder

B. Turbo Decoder

Turbo decoding is done iteratively and the LTE turbo decoder consists of two soft-in-soft-out (SISO) decoders, shown in Figure 2. To compute the extrinsic information, the output of first decoder is fed to the input of second decoder. The turbo decoder has both interleaver and de-interleaver blocks, which helps to reorder the input bits. Each turbo decoder produces *a posteriori* information; this information can be used as *a prior* information to the other decoder. In turbo codes, various decoding algorithms are used to decode the original sequence such as MAP, Max Log MAP [18], and linear version of Max Log MAP algorithm respectively. Here, MAP and Max Log MAP decoding algorithms have been applied and their performance compared. MAP algorithm is also called as BCJR algorithm since it was developed by Bahl, Cocke, Jelinek and Raviv in 1974. Extrinsic information can be calculated using Log Likelihood Ratio (LLR) given by (1),

$$L(u_k) = \ln \left(\frac{p(u_k=+1)}{p(u_k=-1)} \right) \quad (1)$$

Unlike LLR, the conditional LLR $L\left(\frac{u_k}{y}\right)$ is commonly used in decoding algorithms. It is based on the ratio of

a posteriori probabilities to estimate the received sequence, as in (2):

$$L\left(\frac{u_k}{y}\right) = \ln\left(\frac{p(u_k = +\frac{1}{y})}{p(u_k = -\frac{1}{y})}\right), \quad (2)$$

where y is the received codeword. This ratio of the *a posteriori* probabilities will be used by the decoder to provide soft representation of the decoded bits.

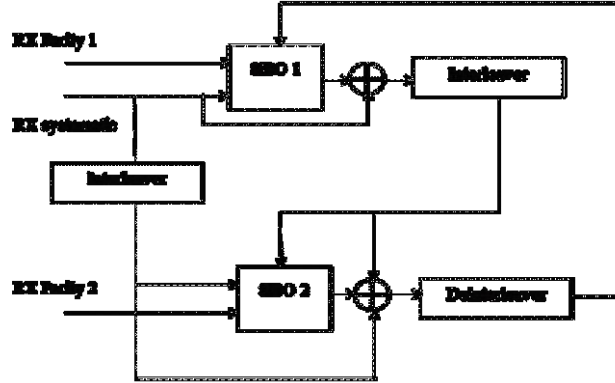


Figure 2. LTE Turbo Decoder

The turbo decoder produces soft output values at the end of each iteration. The soft output from the first component decoder is input to the second component decoder. The principle of BCJR algorithm is to find the state and branch metric values in the trellis diagram. Decoding is mainly based on trellis method to calculate the output values by two metrics. The trellis procedure traces in the backward direction to find the backward state metric and in the forward direction to find the forward state metric. The branch metric and state metric are used to find the LLR.

In MAP decoding algorithm, the LLR is calculated by using (3), (4), (5) and (6).

$$\gamma_k(s', s) = P(y_k/x_k)P(u_k), \quad (3)$$

$$\alpha_k(s) = \sum_{s'} \alpha_{k-1}(s') \gamma_k(s', s), \quad (4)$$

$$\beta_{k-1}(s') = \sum_s \beta_k(s) \gamma_k(s', s), \quad (5)$$

$$L(u_k/y) = \ln \frac{\sum_{(s',s), u_k=1} \alpha_{k-1}(s') \gamma_k(s', s) \beta_k(s)}{\sum_{(s',s), u_k=-1} \alpha_{k-1}(s') \gamma_k(s', s) \beta_k(s)}, \quad (6)$$

α , β , γ represents forward state metric, backward state metric and branch metric values respectively. In MAX Log MAP algorithm, the metrics are calculated using (7), (8), (9) and (10).

$$A_k(s) = \ln(\alpha_k(s)), \quad (7)$$

$$B_k(s) = \ln(\beta_k(s)), \quad (8)$$

$$\Gamma_k(s', s) = \ln(\gamma_k(s', s)), \quad (9)$$

$$L(u_k/y) = \frac{\max_{(s',s), u_k=1} (A_{k-1}(s') + \Gamma_k(s', s)_k + B_k(s))}{\max_{(s',s), u_k=-1} (A_{k-1}(s') + \Gamma_k(s', s)_k + B_k(s))} \quad (10)$$

The MAP decoding algorithm requires more number of additions and multiplications; it increases the computational complexity but it gives acceptable BER performance. MAP algorithm is twice as complex as MAX Log MAP algorithm for storage memory $M=3$. MAX Log MAP algorithm requires lesser number of additions and multiplications because it works in logarithmic domain.

C. Interleaver

The interleaver is an important component in turbo coding, included for better error performance in fading channels. It is mainly used to scramble the input information in a random manner so that errors in strings of consecutive bits are converted to random single bit errors. After scrambling the information, order of the input sequence is changed. In this work, pseudo random interleaver has been used.

IV. CRC AND FLIP AND CHECK ALGORITHM

A. CRC Algorithm

Cyclic Redundancy Check is one of the techniques to detect errors in digitally transmitted data. Generally, error detection is done by introducing redundancy in the original transmitted data. Let there be a message M consisting of k bits. Dividing the original message by generator polynomial G gives a remainder R , so the checksum becomes $(M+R)$. Checksum is appended to the original message M and transmitted over the noisy fading channel. The generator polynomial used for CRC as defined in 3GPP standard is given by (11),

$$g_{CRC24}(D) = [D^{24} + D^{23} + D^6 + D^5 + D + 1], \quad (11)$$

At the end each iteration CRC decoder is used to check for the presence of error. If there is no error it terminates, otherwise it proceeds to the next iteration.

B. Flip and Check Algorithm

In this paper, Flip and Check algorithm is incorporated with CRC to reduce the BER further than that is achieved by turbo codes. The turbo decoder iterates I_{\min} times and then CRC is checked. If CRC is verified, the decoding stops. If not, an extrinsic information based metric Δ_e as given in (12) is computed, that characterizes the reliability of every bit.

$$\Delta_e(k) = |L_{e1}(k) + L_{e2}(k)|, \quad (12)$$

where the extrinsic information Le_1 is from first SISO decoder and Le_2 represents extrinsic information of second SISO decoder.

The least reliable q positions with the least value for the metric Δ_e are extracted and stored. The $(2^q - 1)$ test patterns, with null in positions other than the ones with the least metric value are generated. The non-null bits are flipped and the codewords are generated. For each codeword, if CRC is verified the algorithm stops. Otherwise, the process repeats for the next codeword until all the codewords have been applied or CRC is verified. The above is repeated till the maximum number of iterations I_{\max} is reached.

V. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

OFDM is a modulation technique where multiple orthogonal sub-carriers are used to transmit information. The high bandwidth channel is divided into several narrowband channels. Separate sub-carriers modulate different bit streams that are divided and extracted from the original bit stream. Transmission of the bit streams take place simultaneously. Inter Symbol Interference (ISI) can be reduced by increasing the symbol duration and hence reducing the bit rate. The spectra of sub-carriers are made to overlap for increasing the spectral efficiency. OFDM achieves high data rate transmission with reduced multipath fading, ISI and Inter Channel Interference (ICI).

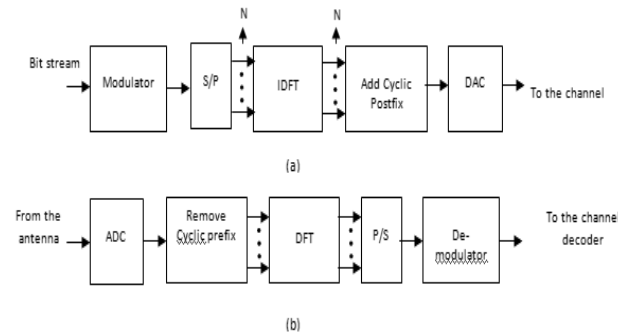


Figure 3. Block Diagram of OFDM (a) Transmitter (b) Receiver

The block diagram of OFDM is given in Figure 3. Each symbol is modulated with the respective sub-carrier using either Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM). The modulated symbols are converted from parallel to serial form before they are transmitted. Adjacent sub-carrier frequency spacing is $2\pi/N$, where N is the number of sub-carriers. The signals are converted from frequency domain to time domain by applying Inverse Discrete Fourier Transform (IDFT) and cyclic prefix is added to the transmitted symbols to avoid ISI. At the receiver, the original signals are recovered by performing inverse operations that were done at the transmitter side. Discrete Fourier Transform (DFT) converts the symbols from time

domain to frequency domain. DFT and IDFT are computed using Fast Fourier and Inverse Fast Fourier Transforms respectively.

Consider the input vector $[X(0), X(1) \dots X(N-1)]$. IFFT can be performed in the transmitter side as in (13),

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} \quad n = 0, 1, 2, \dots (N-1), \quad (13)$$

FFT can be performed in the receiver side as given in (14),

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \quad k = 0, 1, 2, \dots (N-1), \quad (14)$$

Fast Fourier Transform decreases the complexity and increases the performance of the system. In the wireless channel when delay dispersion increases; ISI increases. The last N_{cp} number of samples is copied and appended as a prefix which is known as cyclic prefixing. This acts as a guard interval between OFDM symbols to overcome ISI. It makes the channel equalization simple with the help of Discrete Fourier Transform.

VI. PROPOSED ALGORITHM

The input data is applied to the turbo encoder that encodes the sequence with code rate 1/3. The encoded output from the turbo encoder is appended with 24 bit CRC [19]. The signal is input to the OFDM transmitter and transmitted on the Rayleigh channel. The block diagram of the proposed system is shown in Figure 4.

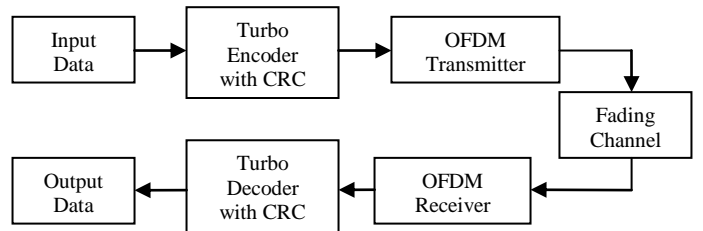


Figure 4. Implementing Turbo Codes in OFDM

VII. RESULTS AND DISCUSSION

In this section, the BER performance of the turbo codes with CRC check for early stopping over AWGN channel and fading channel has been analysed with simulation results. The system model described in Figure 4 is considered for simulation. The binary data from a random source is encoded by the turbo encoder with CRC check and FC as given in Section IV. For simulation, rate 1/3 turbo encoder is used. The frame size is 1024 bits and maximum number of iterations is fixed as 8. The turbo decoder based on MAP and MAX Log MAP algorithms with CRC and FC has been compared. Figure 5 shows the BER performance of Turbo Code using CRC and FC algorithm over AWGN channel.

The graphs reveal that the BER reduces with increase in number of iterations. The performance of MAP algorithm is found to be better compared to Max log Map algorithm.

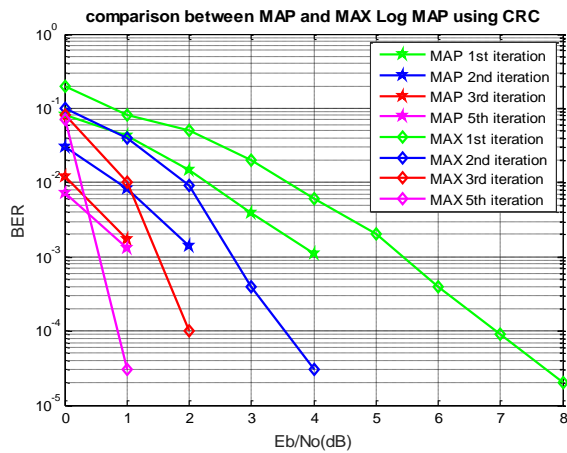


Figure 5. Performance comparison of MAP and MAX-Log MAP decoding algorithms using CRC and FC algorithm

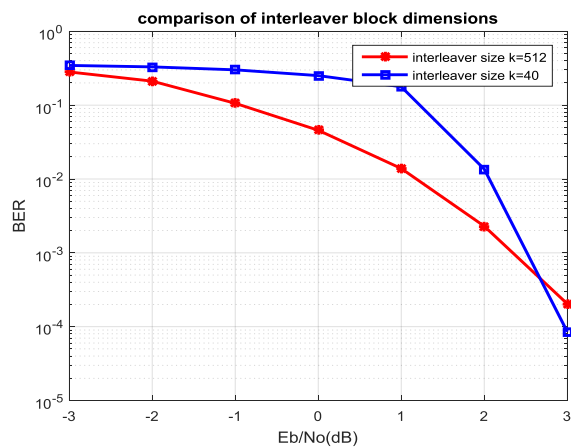


Figure 6. Performance comparison of Interleaver block size before applying CRC and FC algorithm in decoding

Figure 6 and Figure 7 shows the impact of interleaver size on the BER performance with CRC with FC algorithm. Interleaver length of 40 and 512 bits were considered for comparison. The graphs shown in the Figure 6 and Figure 7 reveals that the when the size of the interleaver is 512 bits, 2.3 dB gain at the BER of 10^{-4} could be achieved using CRC and FC algorithm. Increasing the size of the interleaver will randomize the errors present in the received bit sequence, which could be effectively handled by the CRC.

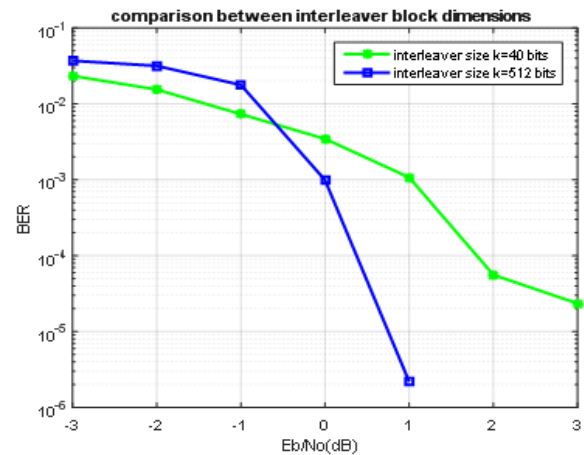


Figure 7. Performances comparison of Interleaver block size using CRC and FC algorithm in decoding

Figure 8 shows the BER performance of Turbo Coded OFDM over Rayleigh Fading channel. After combining the OFDM and turbo codes with CRC with FC algorithm the bit error rate decreases.

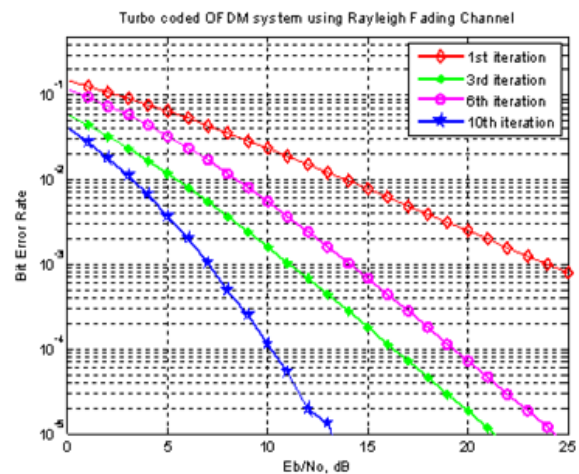


Figure 8. BER performance of Turbo Coded OFDM in Rayleigh Channel

VIII. CONCLUSION

The performance of Turbo Coded OFDM with CRC and FC algorithms is evaluated over Rayleigh fading channel. The error performance of turbo code with CRC and FC using MAP and Max-log-MAP decoding algorithms with different interleaver size has been simulated and analyzed. The simulation results show that increase in interleaver size results in improved performance. Including CRC and FC in turbo encoding has resulted in improved error performance leading to reduced BER. OFDM further improves the system performance due to the inherent multiplexing present. The type of interleaver and choice of decoding algorithm is

important to improve the performance and reduce the complexity in turbo codes.

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