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Effect of AR (2) Model on the Economic Design Mean Chart with Known Coefficient of Variation Under Non Normal Population

J.R. Singh[1], A. Sanvalia[2*]

[1] School of Studies in Statistics,Vikram University, Ujjain, India
[2*] School of Studies in Statistics, Vikram University,Ujjain

Corresponding Author: arvindsanvalia@gmail.com, Tel.: 0-77470-91864

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Abstract— In this paper, we have studied the effect of AR (2) model and non-normality on the economic design of mean chart with known coefficient of variation under non normal population. We use the first four terms of an Edgeworth series for the production cycle time and cost parts. In AR (2) model three different situations arise as (i) Roots are real and distinct (ii) Roots are real and equal (iii) Roots are complex conjugate. The significant effects are seen on mean chart for the above three situation. We also develop the formula for the sample size (n) and sampling interval (h) for different combination of the skewness and kurtosis under AR (2) model.

Keywords— AR (2) Model, Mean Chart, Autocorrelation, Non-Normality, coefficient of variation (CV), Sample Size, Sampling Interval.

I. INTRODUCTION

To evaluate the performance of the $\bar{X}$ control chart, Shewhart (1926), made two main assumptions: the observations are independent and the process is subject to assignable causes shifting the process mean of fits target. A well established measure of statistical performance of a control chart is the Average Run Length (ARL) that represents the expected number of samples taken between the occurrence of the special cause and the signal. An efficient control chart should have a small (large) out-of-control (in-control) ARL. The Shewhart control chart as well as techniques based on the Exponentially Weighted Moving Average (EWMA) and the second order autoregressive models are useful in this regards. A fundamental assumption in the typical application of these control charts is that the observations from different time points are uncorrelated. However, this assumption may not be tenable in some production processes, that is, the continuous chemical process. The presence of autocorrelation in the process data can results in significant effect on the false alarm and detection ability.

In practice, very few quality characteristics of the process are actually normally distributed. With modern measurement and inspection technologies, it is common to collect large volume of data from individual units routinely on very short time intervals. Such nearly continuous measurement inevitably results in data that tend to be non-normally distributed. However, most existing statistical process control (SPC) techniques were not designed for such environments. It is known that conventional SPC techniques are affected by skewed data; especially false alarm rates are so high that true alarms are often ignored. Since the primary purpose of SPC is to detect quickly unusual sources of variability so that their roots causes can be properly addressed, data skewness have serve adverse impacts on the economic benefits of implementing SPC. An efficient control chart implementation requires a correct preliminary selection of its design parameters. Duncan (1956) was the first to present a mathematical model for the design of control charts achieving a pure economic objective based on the minimization of a total cost per hour. This approach does not consider the in-control statistical performance of the control chart: therefore, the control chart implementation can lead to an excessive number of false alarms with a consequent process variability increase due to unnecessary adjustments, see Woodall (1985). Saniga (1989) and Saniga et al. (1995) introduced the economic statistical design of a control chart where statistical constraints limiting the expected number of false alarms are imposed to the cost optimization procedure. The economic design of autocorrelated processes has been investigated by several authors Chou et al.(2001) and Liu et al.(2002) investigated the economic design of $\bar{X}$ control charts for autocorrelated observations, by implementing the Taguchi's quality loss function within the Duncan's economic model; Chen et al.(2007) studied the effect of auto correlation on the economic design of the $\bar{X}$ control chart with variable sample size and sampling interval(VSSI);

Torng et al. (2009) economically designed a $\bar{X}$ control chart with double sampling and used the proposed chart to monitor a real correlated process. Recently Lin et al.(2012) studied the economic design of an ARMA control chart and Franco et al.(2012) used a genetic algorithm to obtain the economically optimal parameters for Shewhart $\bar{X}$ control charts controlling a wandering process mean based on AR(1) model.

In this paper, we have studied the effect of AR (2) model and non-normality on the economic design of mean chart with known coefficient of variation under non normal population. We use the first four terms of an Edgeworth series for the production cycle time and cost parts. In AR (2) model three different situations arise as (i) Roots are real and distinct (ii) Roots are real and equal (iii) Roots are complex conjugate. The significant effects are seen on mean chart for the above three situation. We also develop the formula for the sample size (n) and sampling interval (h) for different combination of the skewness and kurtosis under AR (2) model.

II. MODEL DESCRIPTION OF AR (2) MODEL

Let us define the process whose control will be investigated by

$$x_t = \mu + \xi_t \tag{2.1}$$

Where μ is a constant and ξ_t is a stationary time series with zero mean and standard deviation σ. Suppose that a correlation test revealed that the presence of data dependence and the identification techniques suggested, as the best model, an autoregressive process of order two, AR (2). The AR (2) process can be expressed as

$$\xi_t = \rho_1 \xi_{t-1} + \rho_2 \xi_{t-2} + \varepsilon_t \text{ , t=1, 2, 3, … , n} \tag{2.2}$$

Where

(i) $\xi_t \sim N(0, \sigma_\varepsilon^2)$,

(ii) $$Cov(\varepsilon_t, \varepsilon_\tau) = \begin{cases} \sigma_\varepsilon^2 & ,t = \tau, \quad t = 1, 2, 3, ..., n \\ 0 & ,t \neq \tau, \quad t = 1, 2, 3, ..., n \end{cases} \tag{2.3}$$

An important class of stochastic models for describing time series is the class of stationary models that assume the process to remains in equilibrium about a constant mean level. Stationarity of the AR (2) model it is necessary that the roots of the characteristic equation of AR (2) process by Box and Jenkins (1976) are

$$\phi(B) = 1 - \rho_1 B - \rho_2 B^2 = 0 \tag{2.4}$$

Must lie within the unit disk, which is equivalent to the parameters ρ_1 and ρ_2 must satisfy the following conditions

$$\rho_1 + \rho_2 < 1,\ \rho_2 - \rho_1 < 1 \text{ and } -1 < \rho_2 < 1 \tag{2.5}$$

The variance of AR (2) process is given by

$$\sigma^2 = \frac{(1-\rho_1)}{(1+\rho_2)} \left[\frac{\sigma_\varepsilon^2}{(1-\rho_2)^2 - \rho_1^2} \right] \tag{2.6}$$

Suppose that R_1^{-1} and R_2^{-1} are the roots of the characteristic equation of the process given by equation (2.4) then

$$R_1 = \frac{\rho_1 + \sqrt{\rho_1^2 + 4\rho_2}}{2} \tag{2.7}$$

$$R_2 = \frac{\rho_1 - \sqrt{\rho_1^2 + 4\rho_2}}{2} \tag{2.8}$$

For stationary we require $|R_i| < 1.\ i = 1, 2.$ thus, Singh and Singh (1982) have shown that three situations can theoretically arises

(i) Roots R_1 and R_2 are real and distinct $\rho_1^2 + 4\rho_2 > 0$

(ii) Roots R_1 and R_2 are real and equal $\rho_1^2 + 4\rho_2 = 0$

(iii) Roots R_1 and R_2 are complex conjugate $\rho_1^2 + 4\rho_2 < 0$

We define an estimator for mean under AR (2) process $\bar{x}' = w\sum_{j=i}^{n} x_t$. After some simplification the values of

$$w = \frac{1}{[n + g(\rho_1, \rho_2, n')v^2]} \tag{2.9}$$

$$Min..MSE(\bar{x}') = \frac{\sigma^2}{[n/g(\rho_1, \rho_2, n) + v^2]} \tag{2.10}$$

The probability density function of the sample mean under AR (2) model with known CV is given by

$$f(\bar{x}') = \frac{\sqrt{\frac{n}{g(\rho_1,\rho_2,n)} + v^2}}{\sigma} [\phi\{\frac{\bar{x}' - \mu}{\sigma / \sqrt{\frac{n}{g(\rho_1,\rho_2,n)} + v^2}}\}] \tag{2.11}$$

The error of first kind under AR (2) model with known CV are

$$\alpha = 1 - \int_{\mu - \frac{k\sigma}{\sqrt{n'}}}^{\mu + \frac{k\sigma}{\sqrt{n'}}} f(\bar{x}') d\bar{x}'$$

$$\alpha = 2\Phi[-k\sqrt{\frac{n}{g(\rho_1,\rho_2,n)}+v^2}]$$

$$\alpha = 2\Phi(-kg),$$

where

$$g = \frac{n}{g(\rho_1,\rho_2,n)}+v^2 \tag{2.12}$$

$$\Phi(x) = \int_{-\infty}^{x}\phi(x)dx \tag{2.13}$$

The mean and variance for the distribution of the sample mean $\bar{x}$, when serial correlation is present in the data is given by

$$E(\bar{x}) = \mu$$

and

$$Var(\bar{x}) = \frac{\sigma^2}{n}g(\rho_1,\rho_2,n) = \frac{\sigma^2}{n}g^2. \tag{2.14}$$

Where $g(\rho_1,\rho_2,n))$ depends on the nature of the roots R_1 and R_2, and for different situations is given as follows

(i) R_1 and R_2 are real and distinct,

$$g(\rho_1,\rho_2,n) = \left[\frac{R_1(1-R_2^2)}{(R_1-R_2)(1+R_1R_2)}g(R_1,n) - \frac{R_2(1-R_1^2)}{(R_1-R_2)(1+R_1R_2)}g(R_2,n)\right]$$

$$= g_{rd}(\rho_1,\rho_2,n) \tag{2.15}$$

Where, $g(R,n) = \left[\frac{1+R}{1-R} - \frac{2R}{n}\frac{(1-R^n)}{(1-R)^2}\right]$

(ii) R_1 and R_2 are real and equal,

$$g(\rho_1,\rho_2,n) = \left[\frac{1+R}{1-R} - \frac{2R}{n}\frac{(1-R^n)}{(1-R)^2}\right]\left[1+\frac{(1+R)^2(1-R^n)-n(1-R^2)(1+R^n)}{(1+R^2)(1-R^n)}\right]$$

$$= g_{re}(\rho_1,\rho_2,n) \tag{2.16}$$

(iii) R_1 and R_2 are complex conjugate

$$g(\rho_1,\rho_2,n) = \left[\gamma(d,u) + \frac{2d}{n}(W(d,u,n)+z(d,u,n)\right]$$

$$= g_{cc}(\rho_1,\rho_2,n) \tag{2.17}$$

where,

$$\gamma(d,u) = \frac{1-d^4+2d(1-d^2)\cos u}{(1+d^2)(1+d^2-2d\cos u)}$$

$$W(d,u,n) = \frac{2d(1+d^2)\sin u-(1+d^4)\sin 2u-d^{n+4}\sin(n-2)u}{(1+d^2)(1+d^2-2d\cos u)^2\sin u}$$

$$Z(d,u,n) = \frac{2d^{n+3}\sin(n-1)u-2d^{n+1}\sin(n+1)u+d^n\sin(n+2)u}{(1+d^2)(1+d^2-2d\cos u)^2\sin u}$$

$$d^2 = -\rho_2$$

and

$$u = \cos^{-1}\left(\frac{\rho_1}{2d}\right)$$

The real valued parameters ρ_1 and ρ_2 (the autoregressive parameters) are equal to zero then process is white noise, which is the classical quality control model. If the process is not white noise the quality control factors are quite different from those of the traditional quality factors. How different they are; depending upon the stochastic properties of the AR (2) process.

III. MODEL DESCRIPTION FOR COST FUNCTION

The average net income per hour approximate function of using the control chart for mean of normal variables given by Duncan (1956) is as under:

$$I = V_0 - \frac{\lambda MB+(\alpha T/h)+\lambda W}{1+\lambda B} - \frac{b+cn}{h}, \tag{3.1}$$

where

$V_0 =$ The average income per hour when process is in control and process average is μ.

$V_1 =$ The average income per hour when process is not in control and process average is $\mu' = \mu+\delta\sigma$.

$M = V_0 - V_1$.

$\lambda =$ The average number of times the assignable cause occur within an interval of time.

$B = ah + Cn + D.$

$a = \frac{1}{P} - \frac{1}{2} + \frac{\lambda h}{12}.$

h = Sampling interval in hours.

$C_n =$ The time required to take and inspect a sample of size n.

$D =$ Average time taken to find the assignable cause after a point plotted on the chart falls outside the control limits,

$P =$ Probability of detecting an assignable cause when it exists.

$$P = \int_{-\infty}^{\mu - \frac{k\sigma}{\sqrt{n}}} g(\bar{x}/\mu')d\,\bar{x} + \int_{\mu + \frac{k\sigma}{\sqrt{n}}}^{\infty} g(\bar{x}/\mu')d\,\bar{x}$$

$\cong 1 - \Phi(k - \delta\sqrt{n})$ for $\delta > 0$

where $g(\bar{x}/\mu')$ is the density function of $\bar{X}$ when the true mean μ and $\Phi(x)$ is the normal probability.

$\alpha =$ Probability of wrongly indicating the presence of assignable cause.

$$= \int_{\mu - \frac{k\sigma}{\sqrt{n}}}^{\mu + \frac{k\sigma}{\sqrt{n}}} g(\bar{x}/\mu)d\,\bar{x} = 2\Phi(-k) = \alpha_N$$

$T =$ The cost per occasions of looking for an assignable cause when no assignable cause exists,

$W =$ The average cost per occasion of finding the assignable cause when it exist,

$b =$ Per sample cost of sampling and plotting, that is independent of sample size, and

$c =$ The cost per unit of measuring an item in a sample.

The average cost per hour involved for maintaining the control chart is $\dfrac{(b+cn)}{h}$.

The average net income per hour of the process under the surveillance of the control chart for mean can be rewritten as,

$I = V_0 - L,$

where

$$L = \frac{\lambda MB + (\alpha T/h) + \lambda W}{1 + \lambda B} + \frac{b + cn}{h}. \qquad (3.2)$$

L is considered as the per hour cost due to the surveillance of the process under the control chart. The probability density function for non – normal population is represented by first four terms of Edgeworth series and P' and α' are determined from the sampling distribution of mean and are written as

$$P' = 1 - \Phi(\theta) + \frac{\lambda_3}{6\sqrt{n}}\phi^{(2)}(\theta) - \frac{\lambda_4}{24n}\phi^{(3)}(\theta) - \frac{\lambda_3^2}{72n}\phi^{(5)}(\theta), \quad for\ \delta > 0, \qquad (3.3)$$

where

$\alpha' = \alpha_N - \alpha_c$,

$\alpha_N = 2\Phi(-k)$,

$\theta = (k - \delta\sqrt{n})$, and

$$\alpha_c = \left(\frac{3\lambda_4\phi^{(3)}(k) + \lambda_3^2\phi^{(5)}(k)}{36\,n}\right), \qquad (3.4)$$

is the non- normality correction factor for α.

IV. DERIVATION FOR OPTIMUM VALUE OF SAMPLE SIZE N AND SAMPLING INTERVAL *H*

Determination of the optimum value of sample size n_0 and sampling interval h_0 it can be obtained, either by maximizing the gain function I or by minimizing the cost function L on differentiating the equation (3.2) with respect to *n* and *h* and equating to zero, we get:

$$\frac{\partial L}{\partial n} = \frac{\left\{(1+\lambda B)\left(\lambda M\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha'}{\partial n}\right) - \left(\lambda MB + \frac{\alpha' T}{h} + \lambda W\right)\lambda\frac{\partial B}{\partial n}\right\}}{(1+\lambda B)^2} + \frac{c}{h} = 0, \qquad (4.1)$$

$$\frac{\partial L}{\partial h} = \frac{\left\{(1+\lambda B)\left(\lambda M\frac{\partial B}{\partial h} - \frac{\alpha' T}{h^2}\right) - \left(\lambda MB + \frac{\alpha' T}{h} + \lambda W\right)\lambda\frac{\partial B}{\partial n}\right\}}{(1+\lambda B)^2} - \left(\frac{b+cn}{h^2}\right) = 0, \qquad (4.2)$$

where

$$\frac{\partial B}{\partial n} = \frac{-h}{P'^2}\frac{\partial P'}{\partial n} + c, \qquad (4.3)$$

$$\frac{\partial B}{\partial h} = \frac{1}{P'} - \frac{1}{2} + \frac{\lambda h}{6}, \qquad (4.4)$$

$$\frac{\partial \alpha'}{\partial n} = 0 - \frac{\partial \alpha_c}{\partial n} = \left(\frac{3\lambda_4\phi^{(3)}(k) + \lambda_3^2\phi^{(5)}(k)}{36n^2}\right) = \frac{\alpha_c}{n}, \qquad (4.5)$$

$$\frac{\partial P'}{\partial n} = \frac{\delta}{2\sqrt{n}}\phi(\theta) + \frac{1}{144n^2}\begin{bmatrix} -12\lambda_3\left\{\delta n\phi^3(\theta) - \sqrt{n}\,\phi^2(\theta)\right\} \\ +3\lambda_4\left\{\delta\sqrt{n}\,\phi^4(\theta) + 2\,\phi^3(\theta)\right\} \\ +\lambda_3^2\left\{\delta\sqrt{n}\,\phi^6(\theta) + 2\,\phi^5(\theta)\right\} \end{bmatrix}. \qquad (4.6)$$

The equations (4.1) and (4.2) can be written as:

$$\lambda h\left(M - \lambda MB - \frac{\alpha T}{h} - \lambda W\right)\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha}{\partial n} + \lambda B\left(\lambda M\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha}{\partial n}\right) + c(1+\lambda B)^2 = 0, \qquad (4.7)$$

$$\lambda h^2\left(M - \lambda MB - \frac{\alpha T}{h} - \lambda W\right)\frac{\partial B}{\partial h} - \alpha T(1+\lambda B) + \lambda^2 h^2 MB\frac{\partial B}{\partial n} - (b+cn)(1+\lambda B)^2 = 0. \qquad (4.8)$$

Choosing λ to be small and noting that the optimum *h* is roughly of order of $\dfrac{1}{\sqrt{\lambda}}$, we neglect terms with λB containing λWc, $\dfrac{\alpha' T}{h}$ and the terms equating higher powers of λ. The equations (4.7) and (4.8) are simplified and put in the following form

$$\frac{-\lambda h^2 M}{P'^2}\frac{\partial P'}{\partial n} - \lambda\alpha' T + \frac{T}{h}.\frac{\alpha_c}{n} + c = 0 \qquad (4.9)$$

$$\lambda M h^2\left(\frac{1}{P'}-\frac{1}{2}\right)-(\alpha' T+b+cn)=0\,. \tag{4.10}$$

Solving the equation (4.10) for *h*, we get

$$h=\left\{\frac{\alpha' T+b+cn}{\lambda M\left(\frac{1}{P'}-\frac{1}{2}\right)}\right\}^{\frac{1}{2}}, \tag{4.11}$$

and

$$-\frac{\alpha' T+b+cn}{P'^2\left(\frac{1}{P'}-\frac{1}{2}\right)}\cdot\frac{\partial P'}{\partial n}-\lambda\alpha' T+\frac{T\alpha_c}{n}\left\{\frac{\lambda M\left(\frac{1}{P'}-\frac{1}{2}\right)}{\alpha' T+b+cn}\right\}^{\frac{1}{2}}+c=0\,. \tag{4.12}$$

The values of *n* for which the equation (4.12) satisfy yield us the required optimum value of sample size *n*. Substituting this value *n* in equation (4.11), we find the optimum value of the sampling interval *h*.

V. Derivation for Sample Size n and Sampling Interval h of Economic Design of Mean Chart for AR (2) Model under Non-Normality

Under AR (2) model, the probability density function for non-normal population will be as follows

$$P_e'=1-\Phi(\theta_e)+\frac{\lambda_3 g^3}{6\sqrt{n'}}\phi^{(2)}(\theta_e)-\frac{\lambda_4 g^4}{24n'}\phi^{(3)}(\theta_e)-\frac{\lambda_3^2 g^6}{72n'}\phi^{(5)}(\theta_e) \tag{5.1}$$

$\alpha_e'=\alpha_{Ne}-\alpha_{ce}$, where

$$\theta_e=g(k-\delta\sqrt{n'}\,')$$

$$\alpha_{Ne}=2\Phi(-kg),\quad g^2=g(\rho_1,\rho_2,n) \tag{5.2}$$

$$\alpha_{ce}=\left(\frac{3\lambda_4 g^4\phi^{(3)}(k)+\lambda_3^2 g^6\phi^{(5)}(k)}{36\,n'}\right)$$

$$\frac{\partial P_e'}{\partial n'}=\frac{\delta}{2g\sqrt{n'}}\phi(\theta_e)+\frac{1}{144n'^2}\begin{bmatrix}-12g^3\lambda_3(g\delta n')\phi^3(\theta_e)-\sqrt{n'}\,\phi^2(\theta_e)\\+3\lambda_4 g^4(g\delta\sqrt{n'})\phi^4(\theta_e)+2\,\phi^2(\theta_e)\\+\lambda_3^2 g^6(g\delta\sqrt{n'})\phi^6(\theta_e)+2\,\phi^5(\theta_e)\end{bmatrix} \tag{5.3}$$

For finding the optimum value of sample size n_0 and sampling interval h_0, we get the optimum values of sample size n_0 and sampling interval h_0 of economic design of mean chart for AR (2) model under Non-Normality as

$$h=h_0=\left\{\frac{\alpha_e' T+b+cn'}{\lambda M\left(\frac{1}{P_e'}-\frac{1}{2}\right)}\right\}^{\frac{1}{2}} \tag{5.4}$$

and,

$$-\frac{\alpha_e' T+b+cn'}{P_e'^2\left(\frac{1}{P_e'}-\frac{1}{2}\right)}\cdot\frac{\partial P_e'}{\partial n'}-\lambda\alpha_e' T_e+\frac{T\alpha_{Ce}'}{n'}\left\{\frac{\lambda M\left(\frac{1}{P_e'}-\frac{1}{2}\right)}{\alpha_e' T+b+cn'}\right\}^{\frac{1}{2}}+c=0 \tag{5.5}$$

The values of n_0 for which the equation (5.5) satisfy yield us the required optimum value of sample size n_0. Substituting this value n_0 in equation (5.4), we find the optimum value of the sampling interval h_0 under AR (2) model.

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VI. Numerical Illustration

For the purpose of numerical illustration, we take $\lambda_3=-0.5, 0, 0.5$ and $\lambda_4=-0.5, 0, 1.0, 2.0$, *k*= 2, 2.5, 3, δ= 1.0, 1.5, and 2.0, λ=0.01, *M*=100, *W*=25, *T*=50, *C*=0.05, *D*=2, *b*=0.5, *c* = 0.1. For different values of $(\rho_1,\rho_2)=(0,0), (0.3, 0.6), (0.8, -0.16), (0.8, -0.6)$ determine the optimum values of sample size and sampling interval which are presented in the Tables. We presume the possibility of shift not only in the mean but also in the autoregression parameters (ρ_1,ρ_2). A shift in the autoregression parameters may result from assignable causes occurring over production time, but often also from misidentification of the autoregression model, e.g., a biased estimate of the autoregression parameters. The Table clearly indicates that effect of autocorrelation on sample size n_0 and sampling interval h_0 decrease with increase in *k* and δ when the roots are real and distinct and real and equal. The value of n_0 and h_0 is approximately similar when roots are independent and complex conjugate. For any value of *k* and δ it can be easily seen that effect is maximum on n_0 and h_0 when the roots are real and distinct $(\rho_1=0.3,\rho_2=0.6)$ which is normally the case in practice. From the Table one can observe that, there is not much difference in n_0 and h_0 for normal and non-normal population. From the table it is evident, for a given *k* and δ the value of h_0 increases with the increase in the value of λ_4 and the value of n_0 is the same for different value of *k* and δ. For large value of δ the effect of kurtosis seems to be more marked than that of skewness.

VII. CONCLUSION

Our anxious in this paper has been to explore the time-series suggestion of specific adjustment rules (i.e. n_0 and h_0) to a process in a state of statistical control. Our inspection of these rules revealed that adjustment for n_0 and h_0 under AR (2) process and non-normal process results in systematic time-series behaviour. We have focused completely on the application of specific adjustment factor n_0 and h_0 to a process instead of statistical control. To minimize this effect, the process AR (2) and non-normal could be modernized continuously by using the most recent n_0 values to estimate the value of h_0. Therefore, it is compulsory to point out that the AR (2) and non-normality of the population should be taken into account while designing mean control chart as the optimum values of the control chart parameters are affected by AR (2) and non-normality of the population.

Table-1: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model for independent case $(\rho_1,\rho_2)=(0, 0)$.

$(\rho_1, \rho_2) = (0,0)$		k = 3						k = 2.5						k = 2					
	$\lambda_{3\rightarrow}$	-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
δ	$\lambda_{4\downarrow}$	*n*	*h*	*n*	*h*	*n*	*h*	*n*	*h*	*n*	*h*	*n*	*h*	*n*	*h*	*n*	*h*	*n*	*h*
1.0	**-0.5**	23	2.3279	23	2.3337	23	2.3423	20	2.4076	20	2.4127	19	2.4157	19	3.0305	19	3.0251	19	3.0166
	0.0	23	2.3307	23	2.3371	23	2.3457	20	2.4089	20	2.4134	19	2.4170	19	3.0315	19	3.0258	19	3.0172
	1.0	23	2.3375	23	2.3439	23	2.3525	20	2.4109	20	2.4154	19	2.4190	19	3.0328	19	3.0266	19	3.0177
	2.0	23	2.3442	23	2.3500	23	2.3587	20	2.4123	20	2.4175	19	2.4205	19	3.0341	19	3.0275	19	3.0182
1.5	**-0.5**	11	1.7898	11	1.7912	11	1.7980	9	1.9888	9	1.9916	9	1.9894	9	2.7106	9	2.7044	9	2.6935
	0.0	11	1.8015	11	1.8026	11	1.8092	10	1.9936	9	1.9963	9	1.9938	10	2.7127	9	2.7064	9	2.6947
	1.0	11	1.8240	11	1.8249	11	1.8304	10	2.0031	9	2.0052	9	2.0024	10	2.7175	9	2.7104	9	2.6977
	2.0	11	1.8462	11	1.8465	11	1.8515	10	2.0128	10	2.0146	9	2.0109	10	2.7223	9	2.7146	9	2.7008
2.0	**-0.5**	6	1.5439	6	1.5412	6	1.5477	6	1.8030	6	1.8063	5	1.7996	6	2.5758	6	2.5707	5	2.5582
	0.0	6	1.5677	6	1.5648	6	1.5703	6	1.8130	6	1.8157	5	1.8085	6	2.5796	6	2.5740	5	2.5606
	1.0	7	1.6131	7	1.6091	6	1.6126	6	1.8330	6	1.8346	6	1.8263	6	2.5875	6	2.5812	5	2.5660
	2.0	7	1.6556	7	1.6507	7	1.6526	6	1.8534	6	1.8537	6	1.8440	6	2.5957	6	2.5889	5	2.5724

Table-2: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are Real and Equal for v=2,n=5,10,15 and (ρ_1,ρ_2)=(0.8,-0.16)

(v,n,ρ1,ρ2)	δ	$\lambda_{3\rightarrow}$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_{4\downarrow}$	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(2,5,0.8,-0.16)	1.0	-0.5	15	1.9504	8	0.4943	14	2.1190	5	1.1581	10	1.3529	5	0.8463	3	1.3003	1	0.5831	3	0.9669
		0.0	15	2.1249	15	1.9747	14	2.3133	11	2.0183	11	1.7805	5	1.1371	3	2.0938	8	1.5966	3	1.6095
		1.0	15	2.4154	18	2.3915	14	2.6511	17	2.5152	15	2.3298	11	2.7216	15	2.4866	13	2.2965	8	2.9530
		2.0	22	2.8057	20	2.6836	14	2.9562	19	2.7757	17	2.6505	10	3.1458	17	2.7697	16	2.6433	7	3.5651
	1.5	-0.5	7	1.5249	2	0.3046	6	1.7623	5	1.4369	1	0.6325	2	1.1190	1	4.5110	0	1.5791	3	1.6987
		0.0	10	2.0159	7	1.5251	6	2.3339	9	1.9937	5	1.4153	5	2.5230	1	6.3652	4	1.3144	8	1.9878
		1.0	13	2.4811	11	2.2882	13	2.4811	13	2.4811	11	2.2880	5	3.6400	13	2.4811	11	2.2881	13	2.4811
		2.0	17	2.7696	15	2.6431	17	2.7696	17	2.7696	15	2.6431	5	4.6097	17	2.7696	15	2.6431	17	2.7696
	2.0	-0.5	4	1.3801	1	0.6501	4	1.6981	3	1.3513	1	1.2165	1	2.0907	3	1.1783	0	2.8630	2	1.8770
		0.0	8	1.9875	4	1.3291	8	1.9875	3	2.6020	3	1.2588	1	4.4924	8	1.9876	2	1.1953	8	1.9876
		1.0	13	2.4811	11	2.2880	13	2.4811	13	2.4811	11	2.2880	13	2.4811	13	2.4811	11	2.2881	13	2.4811
		2.0	17	2.7696	15	2.6431	17	2.7696	17	2.7696	15	2.6431	17	2.7696	17	2.7696	15	2.6431	17	2.7696
(2,10,0.8,-0.16)	1.0	-0.5	8	1.4817	14	1.5769	14	2.2800	11	1.8721	3	0.2912	10	2.1811	8	1.7815	1	0.7835	7	2.1533
		0.0	8	1.8161	15	1.9531	14	2.5786	11	2.2358	11	1.7605	10	2.6230	13	2.2415	8	1.5783	7	2.8372
		1.0	9	2.1966	18	2.5272	14	3.0839	18	2.7287	15	2.4835	10	3.3296	17	2.7232	14	2.4705	17	2.7231
		2.0	13	3.0425	21	2.8747	23	3.0375	21	3.0330	19	2.8660	21	3.0329	21	3.0329	19	2.8658	21	3.0329
	1.5	-0.5	6	1.7709	2	0.4575	6	2.1877	7	1.6774	1	0.8610	7	1.6645	6	1.6217	1	1.9279	5	1.5613
		0.0	6	2.4442	7	1.5114	11	2.2143	10	2.2125	5	1.4027	10	2.2125	10	2.2125	4	1.3029	10	2.2125
		1.0	6	3.3858	13	2.4699	17	2.7231	17	2.7231	13	2.4699	17	2.7231	17	2.7231	13	2.4699	17	2.7231
		2.0	6	4.2358	19	2.8658	21	3.0329	21	3.0329	19	2.8658	21	3.0329	21	3.0329	19	2.8658	21	3.0329
	2.0	-0.5	6	1.6100	1	0.8971	6	1.6086	5	1.5848	1	1.5866	5	1.5844	4	1.5754	0	3.3850	4	1.5754
		0.0	10	2.2125	4	1.3194	10	2.2125	10	2.2125	3	1.2499	10	2.2125	10	2.2125	2	1.1872	10	2.2125
		1.0	17	2.7231	13	2.4699	17	2.7231	17	2.7231	13	2.4699	17	2.7231	17	2.7231	13	2.4699	17	2.7231
		2.0	21	3.0329	19	2.8658	21	3.0329	21	3.0329	19	2.8658	21	3.0329	21	3.0329	19	2.8658	21	3.0329
(2,15,0.8,-0.16)	1.0	-0.5	8	2.1428	5	0.1724	8	1.5648	15	2.1081	3	0.4251	10	2.6224	7	2.1827	1	0.9982	7	2.8574
		0.0	8	2.6165	14	1.9354	13	3.0240	16	2.4762	11	1.7438	10	3.2677	14	2.4476	8	1.5632	14	2.4474
		1.0	8	3.0479	19	2.6692	22	2.9722	20	2.9667	16	2.6485	20	2.9667	20	2.9667	16	2.6470	20	2.9667
		2.0	9	3.2326	22	3.0823	26	3.2951	26	3.2951	22	3.0820	26	3.2951	26	3.2951	22	3.0820	26	3.2951
	1.5	-0.5	9	1.9381	2	0.6283	6	3.0960	8	1.9066	1	1.1180	8	1.9062	7	1.8975	1	2.3317	7	1.8975
		0.0	13	2.4446	6	1.5002	13	2.4446	13	2.4446	5	1.3923	13	2.4446	13	2.4446	4	1.2935	13	2.4446
		1.0	20	2.9667	16	2.6470	20	2.9667	20	2.9667	16	2.6470	20	2.9667	20	2.9667	16	2.6470	20	2.9667
		2.0	26	3.2951	22	3.0820	26	3.2951	26	3.2951	22	3.0820	26	3.2951	26	3.2951	22	3.0820	26	3.2951
	2.0	-0.5	7	1.8973	1	1.1811	7	1.8973	7	1.8973	1	2.0089	7	1.8973	7	1.8973	0	4.0122	7	1.8973
		0.0	13	2.4446	4	1.3116	13	2.4446	13	2.4446	3	1.2427	13	2.4446	13	2.4446	2	1.1807	13	2.4446
		1.0	20	2.9667	16	2.6470	20	2.9667	20	2.9667	16	2.6470	20	2.9667	20	2.9667	16	2.6470	20	2.9667
		2.0	26	3.2951	22	3.0820	26	3.2951	26	3.2951	22	3.0820	26	3.2951	26	3.2951	22	3.0820	26	3.2951

Table-3: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are Real and Equal for v=4,n=5,10,15 and (ρ_1,ρ_2)=(0.8,-0.16)

(v,n,ρ1,ρ2)	δ	$\lambda_{3\rightarrow}$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_{4\downarrow}$	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h
(4,5,0.8,-0.16)	1.0	-0.5	36	3.7361	6	1.0606	36	3.7361	36	3.7361	4	1.7208	36	3.7361	36	3.7361	5	0.5914	36	3.7361
		0.0	47	4.1308	13	1.8646	47	4.1308	47	4.1308	9	1.6779	47	4.1308	47	4.1308	7	1.5028	47	4.1308
		1.0	69	4.7262	36	3.7509	69	4.7262	69	4.7262	36	3.7509	69	4.7262	69	4.7262	36	3.7509	69	4.7262
		2.0	89	5.2026	58	4.4595	89	5.2026	89	5.2026	58	4.4595	89	5.2026	89	5.2026	58	4.4595	89	5.2026
	1.5	-0.5	36	3.7361	3	2.4309	36	3.7361	36	3.7361	3	2.3110	36	3.7361	36	3.7361	2	4.0513	36	3.7361
		0.0	47	4.1308	6	1.4561	47	4.1308	47	4.1308	4	1.3516	47	4.1308	47	4.1308	3	1.2567	47	4.1308
		1.0	69	4.7262	36	3.7509	69	4.7262	69	4.7262	36	3.7509	69	4.7262	69	4.7262	36	3.7509	69	4.7262
		2.0	89	5.2026	58	4.4595	89	5.2026	89	5.2026	58	4.4595	89	5.2026	89	5.2026	58	4.4595	89	5.2026
	2.0	-0.5	36	3.7361	1	4.1439	36	3.7361	36	3.7361	1	6.1782	36	3.7361	36	3.7361	1	6.9262	36	3.7361
		0.0	47	4.1308	3	1.2809	47	4.1308	47	4.1308	2	1.2147	47	4.1308	47	4.1308	2	1.1557	47	4.1308
		1.0	69	4.7262	36	3.7509	69	4.7262	69	4.7262	36	3.7509	69	4.7262	69	4.7262	36	3.7509	69	4.7262
		2.0	89	5.2026	58	4.4595	89	5.2026	89	5.2026	58	4.4595	89	5.2026	89	5.2026	58	4.4595	89	5.2026
(4,10,08,-0.16)	1.0	-0.5	42	3.9450	6	1.1967	42	3.9450	42	3.9450	4	1.9195	42	3.9450	42	3.9450	2	3.3919	42	3.9450
		0.0	54	4.3396	13	1.8595	54	4.3396	54	4.3396	9	1.6732	54	4.3396	54	4.3396	6	1.4985	54	4.3396
		1.0	78	4.9535	40	3.8696	78	4.9535	78	4.9535	40	3.8696	78	4.9535	78	4.9535	40	3.8696	78	4.9535
		2.0	101	5.4543	64	4.6151	101	5.4543	101	5.4543	64	4.6151	101	5.4543	101	5.4543	64	4.6151	101	5.4543
	1.5	-0.5	42	3.9450	4	1.4534	42	3.9450	42	3.9450	3	2.8394	42	3.9450	42	3.9450	2	4.6593	42	3.9450
		0.0	54	4.3396	6	1.4530	54	4.3396	54	4.3396	4	1.3488	54	4.3396	54	4.3396	3	1.2541	54	4.3396
		1.0	78	4.9535	40	3.8696	78	4.9535	78	4.9535	40	3.8696	78	4.9535	78	4.9535	40	3.8696	78	4.9535
		2.0	101	5.4543	64	4.6151	101	5.4543	101	5.4543	64	4.6151	101	5.4543	101	5.4543	64	4.6151	101	5.4543
	2.0	-0.5	42	3.9450	2	4.5851	42	3.9450	42	3.9450	1	6.7709	42	3.9450	42	3.9450	1	7.6961	42	3.9450
		0.0	54	4.3396	3	1.2788	54	4.3396	54	4.3396	2	1.2127	54	4.3396	54	4.3396	2	1.1539	54	4.3396
		1.0	78	4.9535	40	3.8696	78	4.9535	78	4.9535	40	3.8696	78	4.9535	78	4.9535	40	3.8696	78	4.9535
		2.0	101	5.4543	64	4.6151	101	5.4543	101	5.4543	64	4.6151	101	5.4543	101	5.4543	64	4.6151	101	5.4543
(4,15,0.8,-0.16)	1.0	-0.5	48	4.1644	6	1.3455	48	4.1644	48	4.1644	4	2.1354	48	4.1644	48	4.1644	2	3.7250	48	4.1644
		0.0	62	4.5616	12	1.8547	62	4.5616	62	4.5616	9	1.6687	62	4.5616	62	4.5616	6	1.4944	62	4.5616
		1.0	89	5.1963	43	3.9912	89	5.1963	89	5.1963	43	3.9912	89	5.1963	89	5.1963	43	3.9912	89	5.1963
		2.0	115	5.7247	71	4.7780	115	5.7247	115	5.7247	71	4.7780	115	5.7247	115	5.7247	71	4.7780	115	5.7247
	1.5	-0.5	48	4.1644	3	3.0109	48	4.1644	48	4.1644	3	3.3750	48	4.1644	48	4.1644	1	7.4854	48	4.1644
		0.0	62	4.5616	6	1.4500	62	4.5616	62	4.5616	4	1.3460	62	4.5616	62	4.5616	3	1.2517	62	4.5616
		1.0	89	5.1963	43	3.9912	89	5.1963	89	5.1963	43	3.9912	89	5.1963	89	5.1963	43	3.9912	89	5.1963
		2.0	115	5.7247	71	4.7780	115	5.7247	115	5.7247	71	4.7780	115	5.7247	115	5.7247	71	4.7780	115	5.7247
	2.0	-0.5	48	4.1644	2	4.3830	48	4.1644	48	4.1644	2	6.0861	48	4.1644	48	4.1644	1	8.4939	48	4.1644
		0.0	62	4.5616	3	1.2767	62	4.5616	62	4.5616	2	1.2109	62	4.5616	62	4.5616	2	1.1523	62	4.5616
		1.0	89	5.1963	43	3.9912	89	5.1963	89	5.1963	43	3.9912	89	5.1963	89	5.1963	43	3.9912	89	5.1963
		2.0	115	5.7247	71	4.7780	115	5.7247	115	5.7247	71	4.7780	115	5.7247	115	5.7247	71	4.7780	115	5.7247

Table-4: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are Real and Equal for v=6,n=5,10,15 and (ρ_1,ρ_2)=(0.8,-0.16)

(v,n,ρ1,ρ2)	δ	λ3→ λ4↓	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
			n	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(6,5,0.8,-0.16)	1.0	-0.5	264	7.9532	7	4.0331	264	7.9532	264	7.9532	7	5.4987	264	7.9532	264	7.9532	4	15.0494	264	7.9532
		0.0	311	8.5286	12	1.8129	311	8.5286	311	8.5286	8	1.6298	311	8.5286	311	8.5286	6	1.4590	311	8.5286
		1.0	405	9.5732	110	5.6278	405	9.5732	405	9.5732	110	5.6278	405	9.5732	405	9.5732	110	5.6278	405	9.5732
		2.0	499	10.5126	203	7.1385	499	10.5126	499	10.5126	203	7.1385	499	10.5126	499	10.5126	203	7.1385	499	10.5126
	1.5	-0.5	264	7.9532	3	8.0646	264	7.9532	264	7.9532	2	11.1855	264	7.9532	264	7.9532	2	13.6827	264	7.9532
		0.0	311	8.5286	5	1.4244	311	8.5286	311	8.5286	4	1.3225	311	8.5286	311	8.5286	3	1.2306	311	8.5286
		1.0	405	9.5732	110	5.6278	405	9.5732	405	9.5732	110	5.6278	405	9.5732	405	9.5732	110	5.6278	405	9.5732
		2.0	499	10.5126	203	7.1385	499	10.5126	499	10.5126	203	7.1385	499	10.5126	499	10.5126	203	7.1385	499	10.5126
	2.0	-0.5	264	7.9532	2	12.4861	264	7.9532	264	7.9532	1	16.7778	264	7.9532	264	7.9532	1	19.2224	264	7.9532
		0.0	311	8.5286	3	1.2593	311	8.5286	311	8.5286	2	1.1950	311	8.5286	311	8.5286	2	1.1382	311	8.5286
		1.0	405	9.5732	110	5.6278	405	9.5732	405	9.5732	110	5.6278	405	9.5732	405	9.5732	110	5.6278	405	9.5732
		2.0	499	10.5126	203	7.1385	499	10.5126	499	10.5126	203	7.1385	499	10.5126	499	10.5126	203	7.1385	499	10.5126
(6,10,0.8,-0.16)	1.0	-0.5	289	8.2742	7	4.2461	289	8.2742	289	8.2742	7	5.8216	289	8.2742	289	8.2742	3	9.5035	289	8.2742
		0.0	339	8.8592	12	1.8111	339	8.8592	339	8.8592	8	1.6281	339	8.8592	339	8.8592	6	1.4574	339	8.8592
		1.0	439	9.9288	116	5.7402	439	9.9288	439	9.9288	116	5.7402	439	9.9288	439	9.9288	116	5.7402	439	9.9288
		2.0	540	10.9006	215	7.3076	540	10.9006	540	10.9006	215	7.3076	540	10.9006	540	10.9006	215	7.3076	540	10.9006
	1.5	-0.5	289	8.2742	3	8.4397	289	8.2742	289	8.2742	3	10.8409	289	8.2742	289	8.2742	2	14.2274	289	8.2742
		0.0	339	8.8592	5	1.4233	339	8.8592	339	8.8592	4	1.3215	339	8.8592	339	8.8592	3	1.2297	339	8.8592
		1.0	439	9.9288	116	5.7402	439	9.9288	439	9.9288	116	5.7402	439	9.9288	439	9.9288	116	5.7402	439	9.9288
		2.0	540	10.9006	215	7.3076	540	10.9006	540	10.9006	215	7.3076	540	10.9006	540	10.9006	215	7.3076	540	10.9006
	2.0	-0.5	289	8.2742	2	12.5580	289	8.2742	289	8.2742	2	15.5802	289	8.2742	289	8.2742	1	19.9249	289	8.2742
		0.0	339	8.8592	3	1.2585	339	8.8592	339	8.8592	2	1.1943	339	8.8592	339	8.8592	2	1.1376	339	8.8592
		1.0	439	9.9288	116	5.7402	439	9.9288	439	9.9288	116	5.7402	439	9.9288	439	9.9288	116	5.7402	439	9.9288
		2.0	540	10.9006	215	7.3076	540	10.9006	540	10.9006	215	7.3076	540	10.9006	540	10.9006	215	7.3076	540	10.9006
(6,15,0.8,-0.16)	1.0	-0.5	318	8.6170	7	4.4722	318	8.6170	318	8.6170	7	6.1577	318	8.6170	318	8.6170	3	9.9145	318	8.6170
		0.0	371	9.2154	12	1.8093	371	9.2154	371	9.2154	8	1.6264	371	9.2154	371	9.2154	6	1.4559	371	9.2154
		1.0	478	10.3150	123	5.8553	478	10.3150	478	10.3150	123	5.8553	478	10.3150	478	10.3150	123	5.8553	478	10.3150
		2.0	585	11.3069	228	7.4859	585	11.3069	585	11.3069	228	7.4859	585	11.3069	585	11.3069	228	7.4859	585	11.3069
	1.5	-0.5	318	8.6170	3	8.8345	318	8.6170	318	8.6170	2	12.1549	318	8.6170	318	8.6170	2	14.7951	318	8.6170
		0.0	371	9.2154	5	1.4222	371	9.2154	371	9.2154	4	1.3205	371	9.2154	371	9.2154	3	1.2288	371	9.2154
		1.0	478	10.3150	123	5.8553	478	10.3150	478	10.3150	123	5.8553	478	10.3150	478	10.3150	123	5.8553	478	10.3150
		2.0	585	11.3069	228	7.4859	585	11.3069	585	11.3069	228	7.4859	585	11.3069	585	11.3069	228	7.4859	585	11.3069
	2.0	-0.5	318	8.6170	2	13.5596	318	8.6170	318	8.6170	1	18.0874	318	8.6170	318	8.6170	1	20.6579	318	8.6170
		0.0	371	9.2154	3	1.2578	371	9.2154	371	9.2154	2	1.1936	371	9.2154	371	9.2154	1	1.1370	371	9.2154
		1.0	478	10.3150	123	5.8553	478	10.3150	478	10.3150	123	5.8553	478	10.3150	478	10.3150	123	5.8553	478	10.3150
		2.0	585	11.3069	228	7.4859	585	11.3069	585	11.3069	228	7.4859	585	11.3069	585	11.3069	228	7.4859	585	11.3069

Table-5: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are Real and Distinct for v=2,n=5,10,15 and (ρ_1,ρ_2)=(0.3,0.6)

(v,n,ρ1,ρ2)	δ	$\lambda_{3\rightarrow}$ / $\lambda_{4\downarrow}$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
			n	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(2,5,0.3,0.6)	1.0	-0.5	15	1.9482	15	1.8144	15	2.0723	11	1.7471	11	1.4930	11	1.8918	8	1.5531	1	0.4745	8	1.7219
		0.0	16	2.0898	15	1.9889	15	2.2211	12	1.9524	12	1.7937	11	2.1136	9	1.8622	8	1.6088	8	2.0660
		1.0	19	2.4580	17	2.3148	15	2.4831	17	2.4040	15	2.2462	11	2.4838	14	2.3640	12	2.2037	8	2.6013
		2.0	21	2.6875	20	2.5840	15	2.7195	18	2.6416	17	2.5364	18	2.6341	16	2.6237	15	2.5178	16	2.6232
	1.5	-0.5	7	1.4634	2	0.2253	7	1.6394	5	1.3397	1	0.5149	5	1.5377	4	1.2118	0	1.4091	4	1.4389
		0.0	10	1.9202	7	1.5341	7	2.0823	5	1.8726	5	1.4237	8	1.8795	7	1.8701	4	1.3221	7	1.8694
		1.0	12	2.3474	10	2.1863	12	2.3473	12	2.3468	10	2.1844	12	2.3468	12	2.3469	10	2.1844	12	2.3469
		2.0	15	2.6217	14	2.5157	15	2.6217	15	2.6217	14	2.5157	15	2.6217	15	2.6218	14	2.5158	15	2.6218
	2.0	-0.5	4	1.2308	1	0.5263	4	1.4508	3	1.1261	1	1.0311	3	1.3731	2	0.9807	0	2.6335	2	1.2662
		0.0	7	1.8667	4	1.3354	7	1.8666	7	1.8658	3	1.2646	7	1.8658	7	1.8659	2	1.2008	7	1.8659
		1.0	12	2.3468	10	2.1843	12	2.3468	12	2.3468	10	2.1843	12	2.3468	12	2.3469	10	2.1844	12	2.3469
		2.0	15	2.6217	14	2.5157	15	2.6217	15	2.6217	14	2.5157	15	2.6217	15	2.6218	14	2.5158	15	2.6218
(2,10,0.3,0.6)	1.0	-0.5	15	1.9469	15	1.7913	15	2.0852	11	1.7504	3	0.1251	5	0.7487	8	1.5635	1	0.5111	3	0.7774
		0.0	15	2.0991	15	1.9837	15	2.2486	12	1.9711	12	1.7891	5	0.9993	8	1.8955	8	1.6045	8	2.1256
		1.0	20	2.4999	18	2.3408	15	2.5348	17	2.4416	15	2.2746	11	2.5572	14	2.4047	12	2.2347	14	2.3998
		2.0	22	2.7268	20	2.6172	14	2.7933	18	2.6860	17	2.5744	18	2.6817	16	2.6727	15	2.5601	16	2.6726
	1.5	-0.5	7	1.4793	2	0.2517	3	0.7788	5	1.3658	1	0.5540	5	1.5858	4	1.2555	0	1.4646	4	1.5106
		0.0	10	1.9522	7	1.5309	7	2.1583	8	1.9220	5	1.4207	5	2.2760	7	1.9087	4	1.3194	7	1.9085
		1.0	12	2.3924	11	2.2205	12	2.3924	12	2.3922	10	2.2196	12	2.3922	12	2.3923	10	2.2197	12	2.3923
		2.0	16	2.6719	14	2.5591	16	2.6718	16	2.6719	14	2.5591	16	2.6718	16	2.6719	14	2.5591	16	2.6719
	2.0	-0.5	4	1.2717	1	0.5672	4	1.5195	3	1.1888	1	1.0923	1	0.4678	1	1.1940	0	2.7048	2	1.4300
		0.0	7	1.9069	4	1.3331	7	1.9069	7	1.9066	3	1.2626	7	1.9066	7	1.9067	2	1.1988	7	1.9067
		1.0	12	2.3922	10	2.2196	12	2.3922	12	2.3922	10	2.2196	12	2.3922	12	2.3923	10	2.2197	12	2.3923
		2.0	16	2.6719	14	2.5591	16	2.6718	16	2.6719	14	2.5591	16	2.6718	16	2.6719	14	2.5591	16	2.6719
(2,15,0.3,0.6)	1.0	-0.5	15	1.9471	15	1.7761	15	2.0949	11	1.7539	11	1.4224	5	0.7775	3	1.1334	1	0.5342	8	1.7684
		0.0	15	2.1063	15	1.9808	15	2.2678	11	1.9846	12	1.7863	5	1.0403	8	1.9193	8	1.6018	8	2.1670
		1.0	20	2.5239	18	2.3571	15	2.5700	17	2.4652	15	2.2925	11	2.6071	14	2.4307	12	2.2545	14	2.4274
		2.0	22	2.7519	20	2.6384	14	2.8430	19	2.7145	17	2.5986	18	2.7114	17	2.7039	15	2.5869	17	2.7038
	1.5	-0.5	7	1.4918	2	0.2686	6	1.6995	5	1.3857	1	0.5790	5	1.6209	4	1.2880	0	1.5006	3	1.5637
		0.0	10	1.9723	7	1.5290	6	2.2108	5	1.9872	5	1.4190	5	2.3496	8	1.9338	4	1.3177	8	1.9336
		1.0	13	2.4209	11	2.2423	13	2.4209	13	2.4208	11	2.2417	13	2.4208	13	2.4209	11	2.2418	13	2.4209
		2.0	16	2.7034	15	2.5863	16	2.7034	16	2.7034	15	2.5863	16	2.7034	16	2.7034	15	2.5863	16	2.7034
	2.0	-0.5	4	1.3023	1	0.5934	4	1.5701	3	1.2350	1	1.1316	1	1.2042	1	0.8678	0	2.7533	1	1.0444
		0.0	7	1.9326	4	1.3318	7	1.9326	7	1.9325	3	1.2613	7	1.9325	7	1.9326	2	1.1976	7	1.9326
		1.0	13	2.4208	11	2.2417	13	2.4208	13	2.4208	11	2.2417	13	2.4208	13	2.4209	11	2.2418	13	2.4209
		2.0	16	2.7034	15	2.5863	16	2.7034	16	2.7034	15	2.5863	16	2.7034	16	2.7034	15	2.5863	16	2.7034

Table-6: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are Real and Distinct for v=4,n=5,10,15 and (ρ_1,ρ_2)=(0.3,0.6)

(v,n,ρ1,ρ2)	δ	$\lambda_{3\rightarrow}$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_{4\downarrow}$	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(4,5,0.3,0.6)	1.0	-0.5	33	3.6237	6	0.9900	33	3.6237	33	3.6237	4	1.6175	33	3.6237	33	3.6237	2	2.9207	33	3.6237
		0.0	44	4.0200	13	1.8674	44	4.0200	44	4.0200	9	1.6806	44	4.0200	44	4.0200	7	1.5054	44	4.0200
		1.0	64	4.6065	35	3.6864	64	4.6065	64	4.6065	35	3.6864	64	4.6065	64	4.6065	35	3.6864	64	4.6065
		2.0	83	5.0699	55	4.3756	83	5.0699	83	5.0699	55	4.3756	83	5.0699	83	5.0699	55	4.3756	83	5.0699
	1.5	-0.5	33	3.6237	3	2.2859	33	3.6237	33	3.6237	3	2.0164	33	3.6237	33	3.6237	2	3.7214	33	3.6237
		0.0	44	4.0200	6	1.4579	44	4.0200	44	4.0200	4	1.3533	44	4.0200	44	4.0200	3	1.2582	44	4.0200
		1.0	64	4.6065	35	3.6864	64	4.6065	64	4.6065	35	3.6864	64	4.6065	64	4.6065	35	3.6864	64	4.6065
		2.0	83	5.0699	55	4.3756	83	5.0699	83	5.0699	55	4.3756	83	5.0699	83	5.0699	55	4.3756	83	5.0699
	2.0	-0.5	33	3.6237	1	3.9122	33	3.6237	33	3.6237	1	5.8647	33	3.6237	33	3.6237	1	14.8634	33	3.6237
		0.0	44	4.0200	3	1.2822	44	4.0200	44	4.0200	2	1.2158	44	4.0200	44	4.0200	2	1.1567	44	4.0200
		1.0	64	4.6065	35	3.6864	64	4.6065	64	4.6065	35	3.6864	64	4.6065	64	4.6065	35	3.6864	64	4.6065
		2.0	83	5.0699	55	4.3756	83	5.0699	83	5.0699	55	4.3756	83	5.0699	83	5.0699	55	4.3756	83	5.0699
(4,10,0.3,0.6)	1.0	-0.5	34	3.6613	6	1.0135	34	3.6613	34	3.6613	4	1.6518	34	3.6613	34	3.6613	2	2.9747	34	3.6613
		0.0	45	4.0571	13	1.8664	45	4.0571	45	4.0571	9	1.6797	45	4.0571	45	4.0571	7	1.5045	45	4.0571
		1.0	65	4.6461	35	3.7082	65	4.6461	65	4.6461	35	3.7082	65	4.6461	65	4.6461	35	3.7082	65	4.6461
		2.0	85	5.1141	56	4.4035	85	5.1141	85	5.1141	56	4.4035	85	5.1141	85	5.1141	56	4.4035	85	5.1141
	1.5	-0.5	34	3.6613	3	2.3341	34	3.6613	34	3.6613	3	2.1163	34	3.6613	34	3.6613	1	6.1223	34	3.6613
		0.0	45	4.0571	6	1.4573	45	4.0571	45	4.0571	4	1.3527	45	4.0571	45	4.0571	3	1.2577	45	4.0571
		1.0	65	4.6461	35	3.7082	65	4.6461	65	4.6461	35	3.7082	65	4.6461	65	4.6461	35	3.7082	65	4.6461
		2.0	85	5.1141	56	4.4035	85	5.1141	85	5.1141	56	4.4035	85	5.1141	85	5.1141	56	4.4035	85	5.1141
	2.0	-0.5	34	3.6613	2	3.0377	34	3.6613	34	3.6613	1	5.9693	34	3.6613	34	3.6613	1	6.6495	34	3.6613
		0.0	45	4.0571	3	1.2818	45	4.0571	45	4.0571	2	1.2154	45	4.0571	45	4.0571	2	1.1563	45	4.0571
		1.0	65	4.6461	35	3.7082	65	4.6461	65	4.6461	35	3.7082	65	4.6461	65	4.6461	35	3.7082	65	4.6461
		2.0	85	5.1141	56	4.4035	85	5.1141	85	5.1141	56	4.4035	85	5.1141	85	5.1141	56	4.4035	85	5.1141
(4,15,03,0.6)	1.0	-0.5	35	3.6853	6	1.0285	35	3.6853	35	3.6853	4	1.6738	35	3.6853	35	3.6853	2	3.0091	35	3.6853
		0.0	46	4.0806	13	1.8658	46	4.0806	46	4.0806	9	1.6791	46	4.0806	46	4.0806	7	1.5039	46	4.0806
		1.0	66	4.6715	36	3.7217	66	4.6715	66	4.6715	36	3.7217	66	4.6715	66	4.6715	36	3.7217	66	4.6715
		2.0	86	5.1423	57	4.4220	86	5.1423	86	5.1423	57	4.4220	86	5.1423	86	5.1423	57	4.4220	86	5.1423
	1.5	-0.5	35	3.6853	3	2.3650	35	3.6853	35	3.6853	3	2.1791	35	3.6853	35	3.6853	2	3.9025	35	3.6853
		0.0	46	4.0806	6	1.4569	46	4.0806	46	4.0806	4	1.3524	46	4.0806	46	4.0806	3	1.2574	46	4.0806
		1.0	66	4.6715	36	3.7217	66	4.6715	66	4.6715	36	3.7217	66	4.6715	66	4.6715	36	3.7217	66	4.6715
		2.0	86	5.1423	57	4.4220	86	5.1423	86	5.1423	57	4.4220	86	5.1423	86	5.1423	57	4.4220	86	5.1423
	2.0	-0.5	35	3.6853	1	4.0386	35	3.6853	35	3.6853	1	6.0360	35	3.6853	35	3.6853	1	6.7382	35	3.6853
		0.0	46	4.0806	3	1.2815	46	4.0806	46	4.0806	2	1.2152	46	4.0806	46	4.0806	2	1.1561	46	4.0806
		1.0	66	4.6715	36	3.7217	66	4.6715	66	4.6715	36	3.7217	66	4.6715	66	4.6715	36	3.7217	66	4.6715
		2.0	86	5.1423	57	4.4220	86	5.1423	86	5.1423	57	4.4220	86	5.1423	86	5.1423	57	4.4220	86	5.1423

Table-7: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are Real and Distinct for v=6,n=5,10,15 and (ρ_1,ρ_2)=(0.3,0.6)

(v,n,ρ1,ρ2)	δ	$\lambda_3\rightarrow$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_4\downarrow$	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(6,5,0.3,0.6)	1.0	-0.5	251	7.7858	10	3.6756	251	7.7858	251	7.7858	7	5.3248	251	7.7858	251	7.7858	3	8.9031	251	7.7858
		0.0	296	8.3550	12	1.8139	296	8.3550	296	8.3550	8	1.6307	296	8.3550	296	8.3550	6	1.4598	296	8.3550
		1.0	387	9.3826	107	5.5682	387	9.3826	387	9.3826	107	5.5682	387	9.3826	387	9.3826	107	5.5682	387	9.3826
		2.0	478	10.3102	197	7.0513	478	10.3102	478	10.3102	197	7.0513	478	10.3102	478	10.3102	197	7.0513	478	10.3102
	1.5	-0.5	251	7.7858	3	7.8644	251	7.7858	251	7.7858	2	10.9322	251	7.7858	251	7.7858	2	13.3896	251	7.7858
		0.0	296	8.3550	5	1.4250	296	8.3550	296	8.3550	4	1.3231	296	8.3550	296	8.3550	3	1.2311	296	8.3550
		1.0	387	9.3826	107	5.5682	387	9.3826	387	9.3826	107	5.5682	387	9.3826	387	9.3826	107	5.5682	387	9.3826
		2.0	478	10.3102	197	7.0513	478	10.3102	478	10.3102	197	7.0513	478	10.3102	478	10.3102	197	7.0513	478	10.3102
	2.0	-0.5	251	7.7858	2	17.2046	251	7.7858	251	7.7858	2	14.6894	251	7.7858	251	7.7858	1	18.8447	251	7.7858
		0.0	296	8.3550	3	1.2597	296	8.3550	296	8.3550	2	1.1954	296	8.3550	296	8.3550	2	1.1385	296	8.3550
		1.0	387	9.3826	107	5.5682	387	9.3826	387	9.3826	107	5.5682	387	9.3826	387	9.3826	107	5.5682	387	9.3826
		2.0	478	10.3102	197	7.0513	478	10.3102	478	10.3102	197	7.0513	478	10.3102	478	10.3102	197	7.0513	478	10.3102
(6,10,03,0.6)	1.0	-0.5	255	7.8437	10	3.7262	255	7.8437	255	7.8437	7	5.3831	255	7.8437	255	7.8437	3	8.9731	255	7.8437
		0.0	301	8.4099	12	1.8135	301	8.4099	301	8.4099	8	1.6304	301	8.4099	301	8.4099	6	1.4595	301	8.4099
		1.0	393	9.4465	108	5.5878	393	9.4465	393	9.4465	108	5.5878	393	9.4465	393	9.4465	108	5.5878	393	9.4465
		2.0	485	10.3781	199	7.0805	485	10.3781	485	10.3781	199	7.0805	485	10.3781	485	10.3781	199	7.0805	485	10.3781
	1.5	-0.5	255	7.8437	3	7.9314	255	7.8437	255	7.8437	3	10.2264	255	7.8437	255	7.8437	2	13.4878	255	7.8437
		0.0	301	8.4099	5	1.4248	301	8.4099	301	8.4099	4	1.3229	301	8.4099	301	8.4099	3	1.2309	301	8.4099
		1.0	393	9.4465	108	5.5878	393	9.4465	393	9.4465	108	5.5878	393	9.4465	393	9.4465	108	5.5878	393	9.4465
		2.0	485	10.3781	199	7.0805	485	10.3781	485	10.3781	199	7.0805	485	10.3781	485	10.3781	199	7.0805	485	10.3781
	2.0	-0.5	255	7.8437	2	11.8840	255	7.8437	255	7.8437	2	14.7939	255	7.8437	255	7.8437	1	18.9713	255	7.8437
		0.0	301	8.4099	3	1.2595	301	8.4099	301	8.4099	2	1.1952	301	8.4099	301	8.4099	2	1.1384	301	8.4099
		1.0	393	9.4465	108	5.5878	393	9.4465	393	9.4465	108	5.5878	393	9.4465	393	9.4465	108	5.5878	393	9.4465
		2.0	485	10.3781	199	7.0805	485	10.3781	485	10.3781	199	7.0805	485	10.3781	485	10.3781	199	7.0805	485	10.3781
(6,15,0.3,0.6)	1.0	-0.5	258	7.8817	7	3.9819	258	7.8817	258	7.8817	7	5.4201	258	7.8817	258	7.8817	3	9.0176	258	7.8817
		0.0	304	8.4510	12	1.8133	304	8.4510	304	8.4510	8	1.6302	304	8.4510	304	8.4510	6	1.4593	304	8.4510
		1.0	396	9.4841	109	5.5998	396	9.4841	396	9.4841	109	5.5998	396	9.4841	396	9.4841	109	5.5998	396	9.4841
		2.0	489	10.4214	200	7.0972	489	10.4214	489	10.4214	200	7.0972	489	10.4214	489	10.4214	200	7.0972	489	10.4214
	1.5	-0.5	258	7.8817	3	7.9740	258	7.8817	258	7.8817	3	10.2783	258	7.8817	258	7.8817	2	13.5503	258	7.8817
		0.0	304	8.4510	5	1.4247	304	8.4510	304	8.4510	4	1.3228	304	8.4510	304	8.4510	3	1.2308	304	8.4510
		1.0	396	9.4841	109	5.5998	396	9.4841	396	9.4841	109	5.5998	396	9.4841	396	9.4841	109	5.5998	396	9.4841
		2.0	489	10.4214	200	7.0972	489	10.4214	489	10.4214	200	7.0972	489	10.4214	489	10.4214	200	7.0972	489	10.4214
	2.0	-0.5	258	7.8817	2	12.3590	258	7.8817	258	7.8817	1	16.6223	258	7.8817	258	7.8817	1	19.0517	258	7.8817
		0.0	304	8.4510	3	1.2595	304	8.4510	304	8.4510	2	1.1952	304	8.4510	304	8.4510	2	1.1383	304	8.4510
		1.0	396	9.4841	109	5.5998	396	9.4841	396	9.4841	109	5.5998	396	9.4841	396	9.4841	109	5.5998	396	9.4841
		2.0	489	10.4214	200	7.0972	489	10.4214	489	10.4214	200	7.0972	489	10.4214	489	10.4214	200	7.0972	489	10.4214

Table-8: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are complex conjugate for v=2,n=5,10,15 and (ρ_1,ρ_2)=(0.8,-0.6)

(v,n,ρ1,ρ2)	δ	$\lambda_{3\rightarrow}$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_{4\downarrow}$	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h
(2,5,0.8,-0.6)	1.0	-0.5	14	2.0683	5	0.1168	14	2.3992	5	2.1867	3	0.3518	10	2.3497	3	4.9011	1	0.8802	7	2.4153
		0.0	14	2.3620	14	1.9445	14	2.7567	10	2.3896	11	1.7525	10	2.8783	13	2.3320	8	1.5709	7	3.2351
		1.0	14	2.8403	19	2.5912	13	3.3532	19	2.8363	16	2.5581	19	2.8361	18	2.8351	14	2.5522	18	2.8351
		2.0	24	3.1547	21	2.9690	13	3.8821	23	3.1538	20	2.9662	23	3.1537	23	3.1538	20	2.9661	23	3.1537
	1.5	-0.5	3	3.9312	2	0.5337	3	2.4067	5	2.0948	1	0.9756	2	7.0078	6	1.7421	1	2.1073	6	1.7416
		0.0	11	2.3188	7	1.5060	11	2.3188	11	2.3187	5	1.3977	2	6.6444	11	2.3187	4	1.2983	11	2.3187
		1.0	18	2.8351	14	2.5521	18	2.8351	18	2.8351	14	2.5521	18	2.8351	18	2.8351	14	2.5521	18	2.8351
		2.0	23	3.1538	20	2.9661	23	3.1537	23	3.1538	20	2.9661	23	3.1537	23	3.1538	20	2.9661	23	3.1537
	2.0	-0.5	3	2.3653	1	1.0232	6	1.7358	6	1.7315	1	1.7746	6	1.7314	6	1.7314	0	3.6625	6	1.7314
		0.0	11	2.3187	4	1.3156	11	2.3187	11	2.3187	3	1.2464	11	2.3187	11	2.3187	2	1.1841	11	2.3187
		1.0	18	2.8351	14	2.5521	18	2.8351	18	2.8351	14	2.5521	18	2.8351	18	2.8351	14	2.5521	18	2.8351
		2.0	23	3.1538	20	2.9661	23	3.1537	23	3.1538	20	2.9661	23	3.1537	23	3.1538	20	2.9661	23	3.1537
(2,10,0.8,-0.6)	1.0	-0.5	13	4.0747	5	0.5239	20	2.7805	17	2.7674	3	0.9284	17	2.7674	17	2.7673	2	1.8195	17	2.7673
		0.0	13	4.9001	13	1.8949	24	3.2029	24	3.2029	10	1.7062	24	3.2029	24	3.2029	7	1.5287	24	3.2029
		1.0	36	3.7477	24	3.1764	36	3.7477	36	3.7477	24	3.1764	36	3.7477	36	3.7477	24	3.1764	36	3.7477
		2.0	47	4.1322	36	3.7310	47	4.1322	47	4.1322	36	3.7310	47	4.1322	47	4.1322	36	3.7310	47	4.1322
	1.5	-0.5	17	2.7673	2	1.3178	17	2.7673	17	2.7673	1	2.1422	17	2.7673	17	2.7673	2	0.9927	17	2.7673
		0.0	24	3.2029	6	1.4749	24	3.2029	24	3.2029	5	1.3690	24	3.2029	24	3.2029	3	1.2723	24	3.2029
		1.0	36	3.7477	24	3.1764	36	3.7477	36	3.7477	24	3.1764	36	3.7477	36	3.7477	24	3.1764	36	3.7477
		2.0	47	4.1322	36	3.7310	47	4.1322	47	4.1322	36	3.7310	47	4.1322	47	4.1322	36	3.7310	47	4.1322
	2.0	-0.5	17	2.7673	1	2.3345	17	2.7673	17	2.7673	1	3.6787	17	2.7673	17	2.7673	1	3.3421	17	2.7673
		0.0	24	3.2029	3	1.2939	24	3.2029	24	3.2029	3	1.2266	24	3.2029	24	3.2029	2	1.1662	24	3.2029
		1.0	36	3.7477	24	3.1764	36	3.7477	36	3.7477	24	3.1764	36	3.7477	36	3.7477	24	3.1764	36	3.7477
		2.0	47	4.1322	36	3.7310	47	4.1322	47	4.1322	36	3.7310	47	4.1322	47	4.1322	36	3.7310	47	4.1322
(2,15,0.8,-0.6)	1.0	-0.5	5	4.0006	6	1.0959	38	3.7912	38	3.7912	4	1.7725	38	3.7912	38	3.7912	2	3.1635	38	3.7912
		0.0	5	5.1588	13	1.8631	49	4.1858	49	4.1858	9	1.6767	49	4.1858	49	4.1858	6	1.5017	49	4.1858
		1.0	71	4.7856	37	3.7827	71	4.7851	71	4.7851	37	3.7827	71	4.7851	71	4.7851	37	3.7827	71	4.7851
		2.0	92	5.2683	60	4.5007	92	5.2683	92	5.2683	60	4.5007	92	5.2683	92	5.2683	60	4.5007	92	5.2683
	1.5	-0.5	38	3.7912	3	2.5034	38	3.7912	38	3.7912	3	2.4526	38	3.7912	38	3.7912	2	4.2126	38	3.7912
		0.0	49	4.1858	6	1.4552	49	4.1858	49	4.1858	4	1.3508	49	4.1858	49	4.1858	3	1.2560	49	4.1858
		1.0	71	4.7851	37	3.7827	71	4.7851	71	4.7851	37	3.7827	71	4.7851	71	4.7851	37	3.7827	71	4.7851
		2.0	92	5.2683	60	4.5007	92	5.2683	92	5.2683	60	4.5007	92	5.2683	92	5.2683	60	4.5007	92	5.2683
	2.0	-0.5	38	3.7912	2	3.3911	38	3.7912	38	3.7912	1	6.3336	38	3.7912	38	3.7912	1	7.1302	38	3.7912
		0.0	49	4.1858	3	1.2803	49	4.1858	49	4.1858	2	1.2142	49	4.1858	49	4.1858	2	1.1552	49	4.1858
		1.0	71	4.7851	37	3.7827	71	4.7851	71	4.7851	37	3.7827	71	4.7851	71	4.7851	37	3.7827	71	4.7851
		2.0	92	5.2683	60	4.5007	92	5.2683	92	5.2683	60	4.5007	92	5.2683	92	5.2683	60	4.5007	92	5.2683

Table-9: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are complex conjugate for v=4,n=5,10,15 and (ρ_1,ρ_2)=(0.8,-0.6)

(v,n,ρ1,ρ2)	δ		k= 3						k= 2.5						k= 2					
		$\lambda_3\rightarrow$	-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_4\downarrow$	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(4,5,0.8,-0.6)	1.0	-0.5	45	4.0446	6	1.2637	45	4.0446	45	4.0446	4	2.0168	45	4.0446	45	4.0446	2	3.5425	45	4.0446
		0.0	58	4.4399	13	1.8572	58	4.4399	58	4.4399	9	1.6711	58	4.4399	58	4.4399	6	1.4967	58	4.4399
		1.0	83	5.0630	10	11.5163	83	5.0630	83	5.0630	41	3.9252	83	5.0630	83	5.0630	41	3.9252	83	5.0630
		2.0	108	5.5764	6	7.5060	108	5.5764	108	5.5764	67	4.6892	108	5.5764	108	5.5764	67	4.6892	108	5.5764
	1.5	-0.5	45	4.0446	3	2.8454	45	4.0446	45	4.0446	3	3.0850	45	4.0446	45	4.0446	2	4.9465	45	4.0446
		0.0	58	4.4399	6	1.4516	58	4.4399	58	4.4399	4	1.3475	58	4.4399	58	4.4399	3	1.2530	58	4.4399
		1.0	83	5.0630	41	3.9252	83	5.0630	83	5.0630	41	3.9252	83	5.0630	83	5.0630	41	3.9252	83	5.0630
		2.0	108	5.5764	67	4.6892	108	5.5764	108	5.5764	67	4.6892	108	5.5764	108	5.5764	67	4.6892	108	5.5764
	2.0	-0.5	45	4.0446	2	4.7995	45	4.0446	45	4.0446	1	7.0568	45	4.0446	45	4.0446	1	15.4960	45	4.0446
		0.0	58	4.4399	3	1.2778	58	4.4399	58	4.4399	2	1.2119	58	4.4399	58	4.4399	2	1.1531	58	4.4399
		1.0	83	5.0630	41	3.9252	83	5.0630	83	5.0630	41	3.9252	83	5.0630	83	5.0630	41	3.9252	83	5.0630
		2.0	108	5.5764	67	4.6892	108	5.5764	108	5.5764	67	4.6892	108	5.5764	108	5.5764	67	4.6892	108	5.5764
(4,10,0.8,-0.6)	1.0	-0.5	77	4.9338	6	1.8966	77	4.9338	77	4.9338	4	2.9249	77	4.9338	77	4.9338	2	4.9177	77	4.9338
		0.0	97	5.3573	12	1.8409	97	5.3573	97	5.3573	9	1.6559	97	5.3573	97	5.3573	6	1.4827	97	5.3573
		1.0	135	6.0811	56	4.3918	135	6.0811	135	6.0811	56	4.3918	135	6.0811	135	6.0811	56	4.3918	135	6.0811
		2.0	174	6.7097	95	5.3269	174	6.7097	174	6.7097	95	5.3269	174	6.7097	174	6.7097	95	5.3269	174	6.7097
	1.5	-0.5	77	4.9338	3	4.1053	77	4.9338	77	4.9338	3	5.1296	77	4.9338	77	4.9338	2	7.3767	77	4.9338
		0.0	97	5.3573	6	1.4416	97	5.3573	97	5.3573	4	1.3382	97	5.3573	97	5.3573	3	1.2447	97	5.3573
		1.0	135	6.0811	56	4.3918	135	6.0811	135	6.0811	56	4.3918	135	6.0811	135	6.0811	56	4.3918	135	6.0811
		2.0	174	6.7097	95	5.3269	174	6.7097	174	6.7097	95	5.3269	174	6.7097	174	6.7097	95	5.3269	174	6.7097
	2.0	-0.5	77	4.9338	2	14.7306	77	4.9338	77	4.9338	1	9.5973	77	4.9338	77	4.9338	1	17.5689	77	4.9338
		0.0	97	5.3573	3	1.2709	97	5.3573	97	5.3573	2	1.2056	97	5.3573	97	5.3573	2	1.1476	97	5.3573
		1.0	135	6.0811	56	4.3918	135	6.0811	135	6.0811	56	4.3918	135	6.0811	135	6.0811	56	4.3918	135	6.0811
		2.0	174	6.7097	95	5.3269	174	6.7097	174	6.7097	95	5.3269	174	6.7097	174	6.7097	95	5.3269	174	6.7097
(4,15,0.8,-0.6)	1.0	-0.5	136	6.0884	7	2.7388	136	6.0884	136	6.0884	4	4.1040	136	6.0884	136	6.0884	2	6.6396	136	6.0884
		0.0	165	6.5726	12	1.8269	165	6.5726	165	6.5726	9	1.6428	165	6.5726	165	6.5726	6	1.4708	165	6.5726
		1.0	224	7.4349	76	4.9154	224	7.4349	224	7.4349	76	4.9154	224	7.4349	224	7.4349	76	4.9154	224	7.4349
		2.0	283	8.2010	135	6.0805	283	8.2010	283	8.2010	135	6.0805	283	8.2010	283	8.2010	135	6.0805	283	8.2010
	1.5	-0.5	136	6.0884	3	5.7142	136	6.0884	136	6.0884	3	7.4053	136	6.0884	136	6.0884	2	10.1011	136	6.0884
		0.0	165	6.5726	5	1.4330	165	6.5726	165	6.5726	4	1.3304	165	6.5726	165	6.5726	3	1.2376	165	6.5726
		1.0	224	7.4349	76	4.9154	224	7.4349	224	7.4349	76	4.9154	224	7.4349	224	7.4349	76	4.9154	224	7.4349
		2.0	283	8.2010	135	6.0805	283	8.2010	283	8.2010	135	6.0805	283	8.2010	283	8.2010	135	6.0805	283	8.2010
	2.0	-0.5	136	6.0884	2	8.7920	136	6.0884	136	6.0884	2	11.1961	136	6.0884	136	6.0884	1	14.6246	136	6.0884
		0.0	165	6.5726	3	1.2651	165	6.5726	165	6.5726	2	1.2003	165	6.5726	165	6.5726	2	1.1429	165	6.5726
		1.0	224	7.4349	76	4.9154	224	7.4349	224	7.4349	76	4.9154	224	7.4349	224	7.4349	76	4.9154	224	7.4349
		2.0	283	8.2010	135	6.0805	283	8.2010	283	8.2010	135	6.0805	283	8.2010	283	8.2010	135	6.0805	283	8.2010

Table-10: Optimum Value of Sample Size n_0 and Sampling Interval h_0 under AR (2) Model when Roots are complex conjugate for v=6,n=5,10,15 and (ρ_1,ρ_2)=(0.8,-0.6)

(v,n,ρ1,ρ2)	δ	$\lambda_3\rightarrow$	k= 3						k= 2.5						k= 2					
			-0.5		0		0.5		-0.5		0		0.5		-0.5		0		0.5	
		$\lambda_4\downarrow$	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
(6,5,0.8,-0.6)	1.0	-0.5	302	8.4280	10	4.2384	302	8.4280	302	8.4280	7	5.9749	302	8.4280	302	8.4280	4	8.4319	302	8.4280
		0.0	354	9.0221	12	1.8103	354	9.0221	354	9.0221	8	1.6273	354	9.0221	354	9.0221	6	1.4567	354	9.0221
		1.0	457	10.1083	119	5.7914	457	10.1083	457	10.1083	119	5.7914	457	10.1083	457	10.1083	119	5.7914	457	10.1083
		2.0	560	11.0832	221	7.3858	560	11.0832	560	11.0832	221	7.3858	560	11.0832	560	11.0832	221	7.3858	560	11.0832
	1.5	-0.5	302	8.4280	3	8.6192	302	8.4280	302	8.4280	3	11.0558	302	8.4280	302	8.4280	1	17.2955	302	8.4280
		0.0	354	9.0221	5	1.4228	354	9.0221	354	9.0221	4	1.3210	354	9.0221	354	9.0221	3	1.2293	354	9.0221
		1.0	457	10.1083	119	5.7914	457	10.1083	457	10.1083	119	5.7914	457	10.1083	457	10.1083	119	5.7914	457	10.1083
		2.0	560	11.0832	221	7.3858	560	11.0832	560	11.0832	221	7.3858	560	11.0832	560	11.0832	221	7.3858	560	11.0832
	2.0	-0.5	302	8.4280	2	13.2605	302	8.4280	302	8.4280	1	17.7232	302	8.4280	302	8.4280	1	20.2590	302	8.4280
		0.0	354	9.0221	3	1.2582	354	9.0221	354	9.0221	2	1.1940	354	9.0221	354	9.0221	1	1.1373	354	9.0221
		1.0	457	10.1083	119	5.7914	457	10.1083	457	10.1083	119	5.7914	457	10.1083	457	10.1083	119	5.7914	457	10.1083
		2.0	560	11.0832	221	7.3858	560	11.0832	560	11.0832	221	7.3858	560	11.0832	560	11.0832	221	7.3858	560	11.0832
(6,10,0.8,-0.6)	1.0	-0.5	431	9.8470	10	5.3697	431	9.8470	431	9.8470	7	7.2855	431	9.8470	431	9.8470	3	11.3192	431	9.8470
		0.0	496	10.4885	11	1.8037	496	10.4885	496	10.4885	8	1.6212	496	10.4885	496	10.4885	6	1.4511	496	10.4885
		1.0	626	11.6649	146	6.2573	626	11.6649	626	11.6649	146	6.2573	626	11.6649	626	11.6649	146	6.2573	626	11.6649
		2.0	757	12.7396	275	8.0913	757	12.7396	757	12.7396	275	8.0913	757	12.7396	757	12.7396	275	8.0913	757	12.7396
	1.5	-0.5	431	9.8470	3	10.1889	431	9.8470	431	9.8470	3	12.8956	431	9.8470	431	9.8470	1	19.7926	431	9.8470
		0.0	496	10.4885	5	1.4188	496	10.4885	496	10.4885	4	1.3174	496	10.4885	496	10.4885	3	1.2260	496	10.4885
		1.0	626	11.6649	146	6.2573	626	11.6649	626	11.6649	146	6.2573	626	11.6649	626	11.6649	146	6.2573	626	11.6649
		2.0	757	12.7396	275	8.0913	757	12.7396	757	12.7396	275	8.0913	757	12.7396	757	12.7396	275	8.0913	757	12.7396
	2.0	-0.5	431	9.8470	2	15.4249	431	9.8470	431	9.8470	2	18.2173	431	9.8470	431	9.8470	1	23.1317	431	9.8470
		0.0	496	10.4885	3	1.2554	496	10.4885	496	10.4885	2	1.1915	496	10.4885	496	10.4885	1	1.1351	496	10.4885
		1.0	626	11.6649	146	6.2573	626	11.6649	626	11.6649	146	6.2573	626	11.6649	626	11.6649	146	6.2573	626	11.6649
		2.0	757	12.7396	275	8.0913	757	12.7396	757	12.7396	275	8.0913	757	12.7396	757	12.7396	275	8.0913	757	12.7396
(6,15,0.8,-0.6)	1.0	-0.5	622	11.6305	10	6.6545	622	11.6305	622	11.6305	7	8.7755	622	11.6305	622	11.6305	3	13.2234	622	11.6305
		0.0	705	12.3244	11	1.7972	705	12.3244	705	12.3244	8	1.6151	705	12.3244	705	12.3244	6	1.4456	705	12.3244
		1.0	871	13.6060	181	6.8081	871	13.6060	871	13.6060	181	6.8081	871	13.6060	871	13.6060	181	6.8081	871	13.6060
		2.0	1038	14.7829	345	8.9263	1038	14.7829	1038	14.7829	345	8.9263	1038	14.7829	1038	14.7829	345	8.9263	1038	14.7829
	1.5	-0.5	622	11.6305	3	12.0351	622	11.6305	622	11.6305	2	16.1174	622	11.6305	622	11.6305	2	19.2463	622	11.6305
		0.0	705	12.3244	5	1.4149	705	12.3244	705	12.3244	4	1.3138	705	12.3244	705	12.3244	3	1.2228	705	12.3244
		1.0	871	13.6060	181	6.8081	871	13.6060	871	13.6060	181	6.8081	871	13.6060	871	13.6060	181	6.8081	871	13.6060
		2.0	1038	14.7829	345	8.9263	1038	14.7829	1038	14.7829	345	8.9263	1038	14.7829	1038	14.7829	345	8.9263	1038	14.7829
	2.0	-0.5	622	11.6305	2	17.9302	622	11.6305	622	11.6305	1	23.3666	622	11.6305	622	11.6305	1	26.4333	622	11.6305
		0.0	705	12.3244	3	1.2528	705	12.3244	705	12.3244	2	1.1891	705	12.3244	705	12.3244	1	1.1330	705	12.3244
		1.0	871	13.6060	181	6.8081	871	13.6060	871	13.6060	181	6.8081	871	13.6060	871	13.6060	181	6.8081	871	13.6060
		2.0	1038	14.7829	345	8.9263	1038	14.7829	1038	14.7829	345	8.9263	1038	14.7829	1038	14.7829	345	8.9263	1038	14.7829

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